PHPE 400
Individual and Group Decision Making

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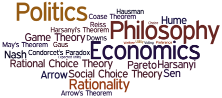


Collective decision making

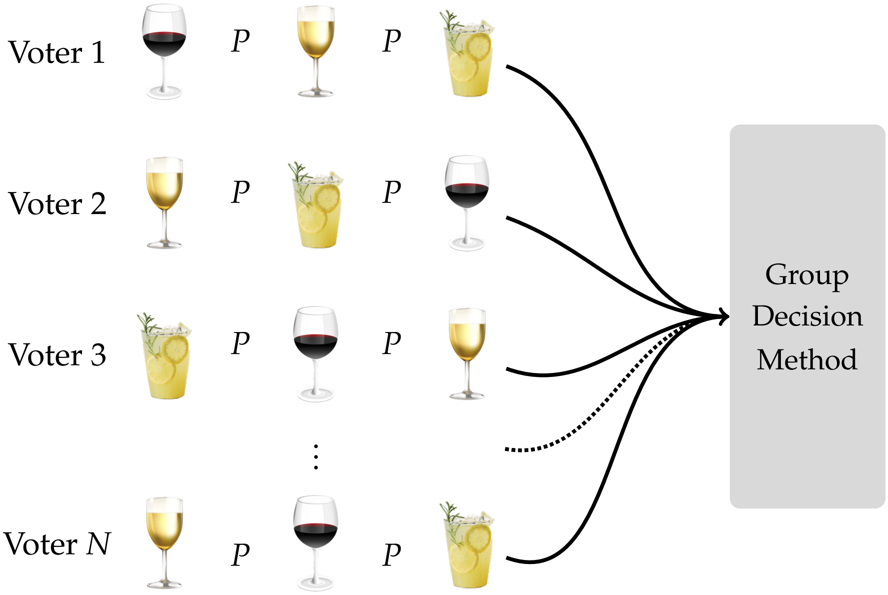
Voter 1  P  P 

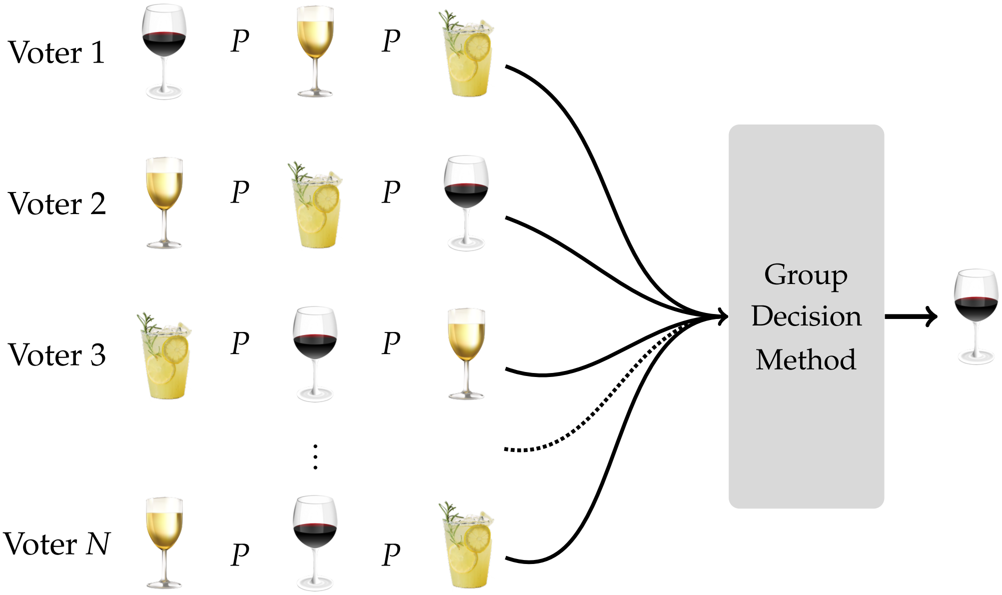
Voter 2  P  P 

Voter 3  P  P 

$\vdots$

Voter N  P  P 


Voter 1
P
P
Voter 2
P
P
Voter 3
P
P
⋮
Voter N
P
P
Group Decision Method


Voter 1
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P
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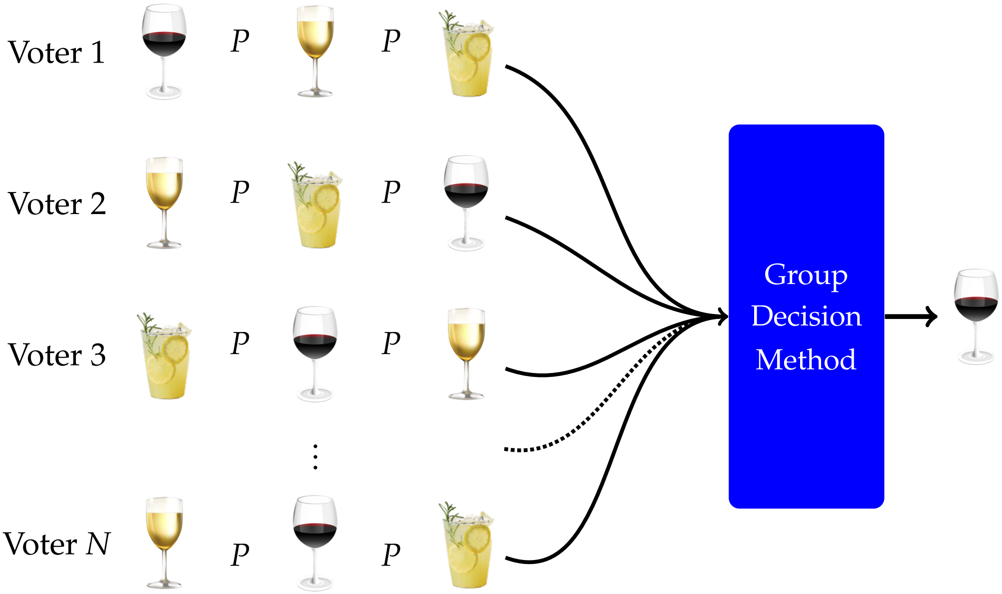
Group Decision Method


Voter 1
P
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Group
Decision
Method

Which candidate *should* be chosen?

40	35	25
t	r	k
k	k	r
r	t	t

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  No candidate has a **majority** of 1st place votes.

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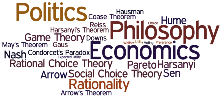

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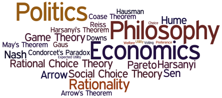
- Candidate r?



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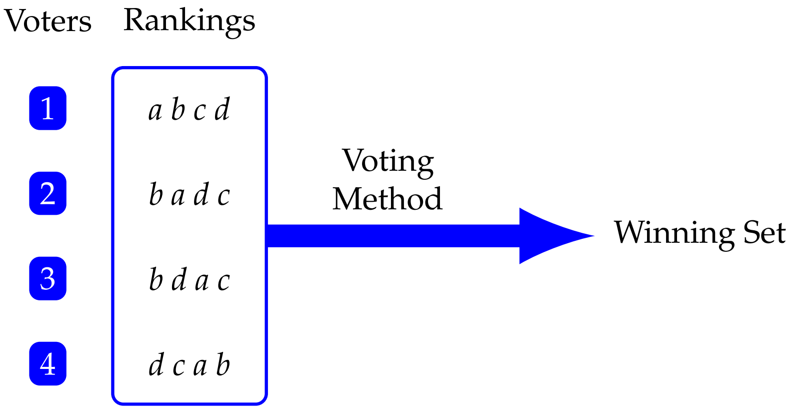
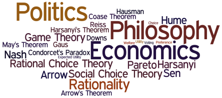
- Candidate k?


Voters
Rankings
1
2
3
4
a b c d
b a d c
b d a c
d c a b
Voting Method
Winning Set

There are many different voting methods



Plurality, Borda Count, Antiplurality/Veto; Coombs; (Strict/Weak) Nanson; Baldwin, Plurality with Runoff; Instant Runoff Voting; Copeland$_\alpha$; Bucklin; Minimax; Beat Path; Split Cycle; Stable Voting; Ranked Pairs; River; GETCHA; GOCHA; Kemeny; Dodgson Method; Young's Method; Approval Voting; Majority Judgment; Cumulative Voting; Range/Score Voting; ...

`https://pref-voting.readthedocs.io/en/latest/collective_decision_procedures.html`

Notation

- V is a finite set of voters (assume that $V = \{1, 2, 3, \ldots, n\}$)
- X is a (typically finite) set of alternatives, or candidates
- An election **profile** is a record of the **ballot** submitted by each voter, where a ballot can be any of the following:
  - A selected candidate
  - A ranking of the candidates
  - Scores/grades assigned to each candidate

Rankings

CITY AND COUNTY 市縣						
MAYOR **市長**	**1** 1st Choice 第一選擇	**2** 2nd Choice 第二選擇	**3** 3rd Choice 第三選擇	**4** 4th Choice 第四選擇	**5** 5th Choice 第五選擇	**6** 6th Choice 第六選擇
ELLEN LEE ZHOU / 李愛晨 Behavioral Health Clinician 行為健康臨床治療師	● 1	2	3	4	5	6
LONDON N. BREED / 倫敦 · 布里德 Mayor of San Francisco 三藩市市長	1	2	● 3	4	5	6
JOEL VENTRESCA / 喬爾 · 范崔斯卡 Retired Airport Analyst 退休機場分析師	1	2	3	4	● 5	6
WILMA PANG / 彭德懸 Retired Music Professor 退休音樂教授	1	2	3	● 4	5	6
ROBERT L. JORDAN, JR. / 小羅伯特 · L · 喬丹 Preacher 傳教士	1	2	3	4	5	● 6
PAUL YBARRA ROBERTSON / 保羅 · 伊巴拉 · 羅伯森 Small Business Owner 小企業業主	1	● 2	3	4	5	6
	1	2	3	4	5	6

Rankings

Let X be a set of candidates and V a set of voters.

A **ranking** of X is a strict linear order P on X: a relation $P \subseteq X \times X$ satisfying the following conditions for all $x, y, z \in X$:

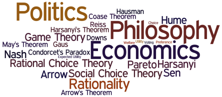
> *asymmetry*: if $x \ P \ y$ then *not* $y \ P \ x$;
>
> *transitivity*: if $x \ P \ y$ and $y \ P \ z$, then $x \ P \ z$;
>
> *weak completeness*: if $x \neq y$, then $x \ P \ y$ or $y \ P \ x$.

Let $L(X)$ be the set of all strict linear orders on X.

Profiles

A **profile** for X is a function $\mathbf{P}$ assigning to $i \in V$ a linear order $\mathbf{P}_i$ on X.

Profiles



A **profile** for X is a function $\mathbf{P}$ assigning to $i \in V$ a linear order $\mathbf{P}_i$ on X.

So, $a \; \mathbf{P}_i \; b$ means that voter i ranks a above b, or that i strictly prefers candidate a to b.

For instance,

Example: let $V = \{v_1, v_2, v_3, v_4\}$ and $X = \{a, b, c, d\}$ and consider the following profile $\mathbf{P}$,

v_1	v_2	v_3	v_4
a	a	b	c
b	c	a	b
c	b	c	a

E.g., $a \; \mathbf{P}_{v_2} \; c$, $b \; \mathbf{P}_{v_4} \; a$, $a \; \mathbf{P}_{v_1} \; b$, $\ldots$

Anonymous Profiles

2	5	3	5
a	a	b	c
b	c	a	b
c	b	c	a

(Linear) Profiles

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}
b	b	b	b	b	b	b	a	a	a	a	a	a	a	a
c	c	c	c	c	c	c	c	c	c	c	c	b	b	b
a	a	a	a	a	a	a	b	b	b	b	b	c	c	c

(Linear) Anonymous Profile

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}
b	b	b	b	b	b	b	a	a	a	a	a	a	a	a
c	c	c	c	c	c	c	c	c	c	c	c	b	b	b
a	a	a	a	a	a	a	b	b	b	b	b	c	c	c

7	5	3
b	a	a
c	c	b
a	b	c

Important Distinction

1	1	1
a	a	d
b	c	a
c	d	b
d	b	c

Do all of the voters rank a and b in the same way?

Do all of the voters rank a and b in the same *position*?

Important Distinction

1	1	1
a	*a*	*d*
b	*c*	*a*
c	*d*	*b*
d	*b*	*c*

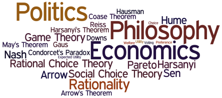
Do all of the voters rank *a* and *b* in the same way?

Yes: All of the voters rank *a* above *b*.

Do all of the voters rank *a* and *b* in the same *position*?

No: The first group ranks *a* in first-place and *b* in second-place, the second group ranks *a* in first-place and *b* is last place, and the third group ranks *a* is second-place and *b* in third-place.

Voting Method



A **voting method** is a function that assigns a set of candidates (the winning set) to a profile.

Formally, a voting method is $F : L(X)^V \to \wp(X) \setminus \{\varnothing\}$, where $L(X)^V$ is the set of profiles of linear orders over X.

Majoritarianism

When there are only **two** candidates a and b, then all (reasonable) voting methods give the same results:

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Yes. We will look at two arguments: A procedural justification and an epistemic justification.

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Results are more mixed: Consider our previous definition of majority rule....

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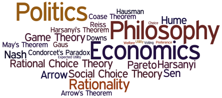

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The problem is that with more than 2 candidates, there may not be any candidate that is ranked first by more than half of the voters.

Positional scoring rules

A **scoring rule** each voter submits a ranking of the candidates. Based on the ranking, each voter assigns a *score* to each candidate. The candidates overall score is the sum of the scores assigned to the candidate by each voter. Then, the candidate(s) with the greatest overall score is(are) the winner(s).

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- **Plurality**: Each voter assigns a score of 1 to the candidate ranked in first place and 0 to all other candidates.

- **Borda**: If there are n candidates, then each voter assigns a score of $n - 1$ to the candidate in first place, $n - 2$ to the candidate in 2nd place, ..., and 0 to the candidate in last place.

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b

	7	5	4	3
1	a	b	d	c
0	b	c	b	d
0	c	d	c	a
0	d	a	a	b

Plurality winner(s): a

$$
\begin{array}{lccccccccc}
\text{Plurality score of } a: & 1*7 & + & 0*0 & + & 0*3 & + & 0*9 & = & 7 \\
\text{Plurality score of } b: & 1*5 & + & 0*11 & + & 0*0 & + & 0*3 & = & 5 \\
\text{Plurality score of } c: & 1*4 & + & 0*5 & + & 0*11 & + & 0*0 & = & 4 \\
\text{Plurality score of } d: & 1*3 & + & 0*3 & + & 0*5 & + & 0*7 & = & 3
\end{array}
$$

	7	5	4	3
3	a	b	d	c
2	b	c	b	d
1	c	d	c	a
0	d	a	a	b

Borda winner(s): b

$$\begin{array}{lcccccccccc}
\text{Borda score of } a\text{:} & 3*7 & + & 2*0 & + & 1*3 & + & 0*9 & = & 24 \\
\text{Borda score of } b\text{:} & 3*5 & + & 2*11 & + & 1*0 & + & 0*3 & = & 37 \\
\text{Borda score of } c\text{:} & 3*4 & + & 2*5 & + & 1*11 & + & 0*0 & = & 33 \\
\text{Borda score of } d\text{:} & 3*3 & + & 2*3 & + & 1*5 & + & 0*7 & = & 20
\end{array}$$

1	2	2
x	y	y
y	x	x

Who are the Borda winners? y

1	2	2
x	y	y
a_1	x	x
a_2	a_1	a_1
a_3	a_2	a_2
y	a_3	a_3

Who are the Borda winners?

1	2	2
x	y	y
a_1	x	x
a_2	a_1	a_1
a_3	a_2	a_2
y	a_3	a_3

Who are the Borda winners? x and y

1	2	2
x	y	y
a_1	x	x
a_2	a_1	a_1
a_3	a_2	a_2
a_4	a_3	a_3
y	a_4	a_4

Who are the Borda winners?

1	2	2
x	y	y
a_1	x	x
a_2	a_1	a_1
a_3	a_2	a_2
a_4	a_3	a_3
y	a_4	a_4

Who are the Borda winners? x,

but a majority of voters prefer y over x.