PHPE 400
Individual and Group Decision Making

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Nash Equilibria

Row \ Column	l	r
u	$1, 1$	$2, 1$
d	$1, 2$	$3, 3$

Pure Strategy Nash Equilibria (u, l) and (d, r)

Nash Equilibria

Row \ Column	l	r
u	$1, 1$	$2, 1$
d	$1, 2$	$3, 3$

Pure Strategy Nash Equilibria (u, l) and (d, r)

Row \ Column	l	r
u	$2, 2$	$2, 1$
d	$1, 3$	$10, 2$

Pure Strategy Nash Equilibria (u, l)

Nash Equilibria

Row \ Column	l	r
u	$1, 1$	$2, 1$
d	$1, 2$	$3, 3$

Pure Strategy Nash Equilibria (u, l) and (d, r)

Row \ Column	l	r
u	$2, 2$	$2, 1$
d	$1, 3$	$10, 2$

Pure Strategy Nash Equilibria (u, l)

Row \ Column	l	r
u	$0, 1$	$2, 1$
d	$2, 0$	$2, 2$

Pure Strategy Nash Equilibria (u, r) and (d, r)

Why play a Nash equilibrium?

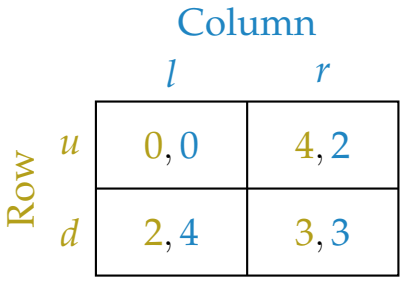
Self-Enforcing Agreements: Nash equilibria are recommended by being the only strategy combinations on which the players could make self-enforcing agreements, i.e., agreements that each has reason to respect, even without external enforcement mechanisms.

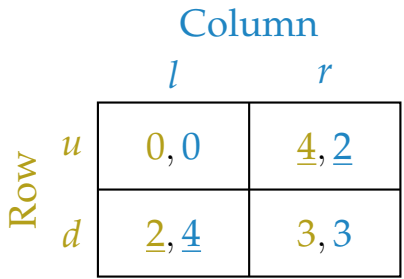
M. Risse (2000). *What is rational about Nash equilibria?*. Synthese, 124:3, pp. 361 - 384.

Row \ Column	l	c	r
u	$4, 6$	$5, 4$	$0, 0$
m	$5, 7$	$4, 8$	$0, 0$
d	$0, 0$	$0, 0$	$1, 1$

Row \ Column	l	c	r
u	$4, \underline{6}$	$\underline{5}, 4$	$0, 0$
m	$\underline{5}, 7$	$4, \underline{8}$	$0, 0$
d	$0, 0$	$0, 0$	$\underline{1}, \underline{1}$

(d, r) is a Nash equilibrium, but it is **not self-enforcing**


Column
l
r
Row
u
d
0, 0
4, 2
2, 4
3, 3



(d, r) is **not** a Nash equilibrium, but it is **self-enforcing**

Self-Enforcing Agreements: Nash equilibria are recommended by being the only strategy combinations on which the players could make self-enforcing agreements, i.e., agreements that each has reason to respect, even without external enforcement mechanisms.

- There are Nash equilibria that are not self-enforcing
- There are self-enforcing outcomes that are not Nash equilibria

Is a Nash equilibrium *guaranteed* by players that are rational and have *common knowledge* of each others' rationality?

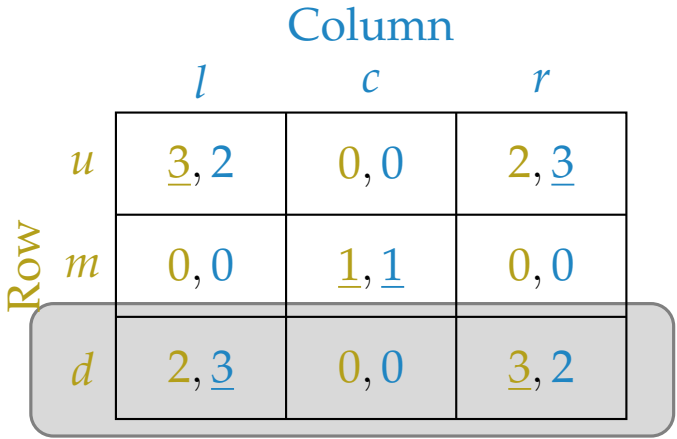
Row \ Column	l	c	r
u	$3, 2$	$0, 0$	$2, 3$
m	$0, 0$	$1, 1$	$0, 0$
d	$2, 3$	$0, 0$	$3, 2$

Row \ Column	l	c	r
u	$\underline{3}, 2$	$0, 0$	$2, \underline{3}$
m	$0, 0$	$\underline{1}, \underline{1}$	$0, 0$
d	$2, \underline{3}$	$0, 0$	$\underline{3}, 2$

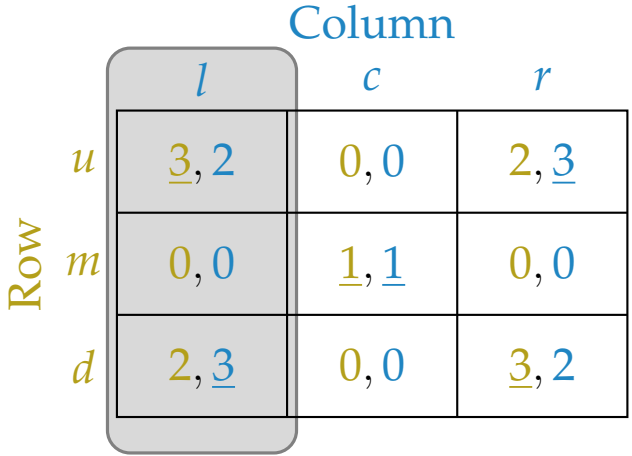
(m, c) is the unique Nash equilibrium

Row \ Column	l	c	r
u	$\underline{3}, 2$	$0, 0$	$2, \underline{3}$
m	$0, 0$	$\underline{1}, \underline{1}$	$0, 0$
d	$2, \underline{3}$	$0, 0$	$\underline{3}, 2$

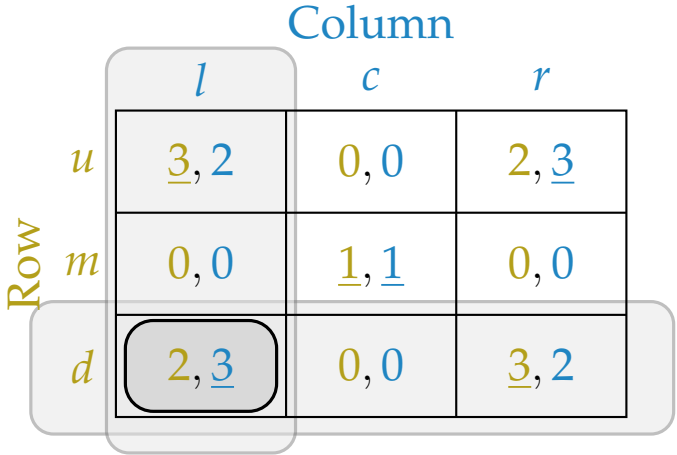
u, d, l, and r are all **rationalizable**



Row plays d because she thought Column would play r



Column plays l because she thought Row would play d



Column was correct, but Row was wrong.
Both players are **rational**.

Row \ Column	l	c	r	x
u	$\underline{3}, 2$	$0, 0$	$2, \underline{3}$	$0, -5$
m	$0, 0$	$\underline{1}, \underline{1}$	$0, 0$	$\underline{100}, -5$
d	$2, \underline{3}$	$0, 0$	$\underline{3}, 2$	$1, -3$

Not every strategy is rationalizable

Row \ Column	l	c	r	x
u	$\underline{3}, 2$	$0, 0$	$2, \underline{3}$	$0, -5$
m	$0, 0$	$\underline{1}, \underline{1}$	$0, 0$	$\underline{100}, -5$
d	$2, \underline{3}$	$0, 0$	$\underline{3}, 2$	$1, -3$

Not every strategy is rationalizable:
Row can't play m *because* she thinks Column will play x

An action a **strictly dominates** another action b for player i when i's utility is strictly better choosing a than choosing b no matter what actions are chosen by the other players.

Example

Row \ Column	l	r
u	$5, 5$	$-100, 4$
d	$0, 1$	$0, 0$

Example

Row \ Column	l	r
u	$5, 5$	$-100, 4$
d	$0, 1$	$0, 0$

Since r is **strictly dominated** by l, Column will not play r.
Then, the best response for Row is u.

Important Games

Coordination

Row \ Column	a	b
a	$\underline{1}, \underline{1}$	$0, 0$
b	$0, 0$	$\underline{1}, \underline{1}$

- Both (a, a) and (b, b) are Nash equilibria
- Both (a, a) and (b, b) are Pareto optimal
- The players want to coordinate by choosing the *same* action a or b.

Anti-Coordination

Row \ Column	a	b
a	$0, 0$	$\underline{1}, \underline{1}$
b	$\underline{1}, \underline{1}$	$0, 0$

- Both (a, b) and (b, a) are Nash equilibria
- Both (a, b) and (b, a) are Pareto optimal
- The players want to mis-coordinate in which one player chooses a and the other chooses b.

Coordination and Competition

Row \ Column	a	b
a	$0, 0$	$\underline{2}, \underline{1}$
b	$\underline{1}, \underline{2}$	$0, 0$

- Both (a, b) and (b, a) are Nash equilibria
- Both (a, b) and (b, a) are Pareto optimal
- Players want to mis-coordinate, and both prefer choosing b while the other chooses a.

Cailin O'Connor (2019). *The Origins of Unfairness: Social Categories and Cultural Evolution*. Oxford University Press.