PHPE 400
Individual and Group Decision Making

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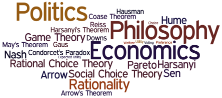
Just Enough Game Theory



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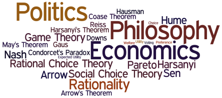

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- the actions the players *can* take

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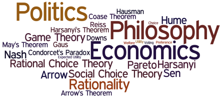

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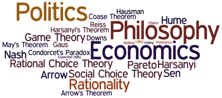

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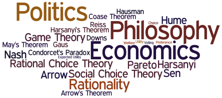

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- the group of players in the game
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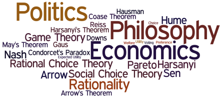
- the players' interests (i.e., preferences/utilities),
- the "structure" of the decision problem (what information do the players have?, what order do they act in?, how many times do they repeat their interaction?, etc.)

*It does **not** specify the actions that the players **do take**.*

Simultaneous-move

In **simultaneous-move games**, also called **strategic games** or **normal form games**, all players select an action simultaneously, without knowing what the others will do (though they can certainly *reason* about what the other players are expected to do).

Strategic Games



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- N is a finite set of **players**
- for each $i \in N$, A_i is a nonempty set of **actions** (also called **strategies**)
- for each $i \in N$, u_i is a **utility function** for player i on the set of outcomes (called strategy profiles): $u_i : \times_{i \in N} A_i \to \mathbb{R}$.

Strategic Games: Example

		Column	
		l	r
Row	u	$2, 1$	$0, 0$
	d	$0, 0$	$1, 2$

- $N = \{Row, Column\}$
- $A_{Row} = \{u, d\}$, $A_{Column} = \{l, r\}$
- $u_{Row} : A_{Row} \times A_{Column} \to \mathbb{R}$, $u_{Column} : A_{Row} \times A_{Column} \to \mathbb{R}$ with $u_{Row}(u, l) = u_{Column}(d, r) = 2$, $u_{Row}(d, r) = u_{Column}(u, l) = 1$, and $u_{Row}(d, l) = u_{Column}(d, l) = u_{Row}(u, r) = u_{Column}(u, r) = 0$.

Strategy Profiles

		Column	
		l	r
Row	u	$2, 1$	$0, 0$
	d	$0, 0$	$1, 2$

A **strategy profile** is a list of actions, one for each player, that represents the outcome of the game.

The 4 possible strategy profiles in the above game are

$$\{(u, l), (d, l), (u, r), (d, r)\}$$

Important Point

The goal of the players is to maximize **their own utility**. The players' utilities represent all of their opinions about the outcome of the game (e.g., "winning the game" or "beating the other player").

Pareto

A strategy profile s **Pareto dominates** a strategy profile t provided *every* player strictly prefers the outcome in s than the outcome in t.

For example, with two players, a strategy profile (x, y) **Pareto dominates** a strategy profile (x', y') when

$$u_1(x, y) > u_1(x', y') \text{ and } u_2(x, y) > u_2(x', y').$$

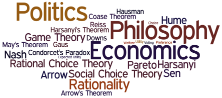
A strategy profile s is **Pareto optimal** if there is no other profile that Pareto dominates s.

Pareto

		Column a	Column b
Row	a	$3, 3$	$0, 0$
Row	b	$0, 0$	$1, 1$

- (a, a) Pareto dominates (a, b)
- (a, a) Pareto dominates (b, a), and
- (a, a) Pareto dominates (b, b).
- (a, a) is the only Pareto optimal profile.

Pareto



Row \ Column	a	b
a	2, 2	1, 3
b	3, 1	0, 0

- (a, a) Pareto dominates (b, b)
- (a, b) Pareto dominates (b, b), and
- (b, a) Pareto dominates (b, b).
- (a, a), (a, b), and (b, a) are all Pareto optimal.

Pareto

		Column	
		a	b
Row	a	$1, -1$	$-1, 1$
	b	$-1, 1$	$1, -1$

- All strategy profiles are Pareto optimal

Solution Concept

A **solution concept** is a systematic description of the outcomes (i.e., the strategy profiles) that may emerge in a family of games.

This is the starting point for most of game theory and includes many variants.

These are usually thought of as the embodiment of “rational behavior” in some way and used to analyze game situations.

Best Response

The **best response** for player i to a list of the other players' actions is the action that maximizes i's utility *assuming that the other players choose their action in the list*.

Best Response

		Column	
		l	r
Row	u	2, 1	0, 0
	d	0, 0	1, 2

Row: The best response to l is u and the best response to r is d

Best Response

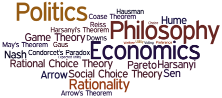
Column

Row		l	r
	u	$\underline{2}, \underline{1}$	$0, 0$
	d	$0, 0$	$\underline{1}, \underline{2}$

Row: The best response to l is u and the best response to r is d

Column: The best response to u is l and the best response to d is r

Nash Equilibrium



A strategy profile is a **Nash equilibrium** if every player's strategy is a best response to the other player's strategies.

Nash Equilibrium: Example

		Column	
		l	r
Row	u	$\underline{2}, \underline{1}$	$0, 0$
	d	$0, 0$	$\underline{1}, \underline{2}$

(u, l) is a Nash Equilibrium

(d, r) is a Nash Equilibrium

Matching Pennies

		Column	
		l	r
Row	u	$\underline{1}, -1$	$-1, \underline{1}$
	d	$-1, \underline{1}$	$\underline{1}, -1$

There are no pure strategy Nash equilibria.

Mixed Strategies

A **mixed strategy** is a probability distribution over the set of pure strategies.

For instance, if a and b are the available actions, then the following are examples of mixed strategies:

- $1/2 \cdot a + 1/2 \cdot b$
- $1/3 \cdot a + 2/3 \cdot b$
- $4/5 \cdot a + 1/5 \cdot b$
- $\ldots$

Matching Pennies

Column

		l	r
Row	u	$\underline{1}, -1$	$-1, \underline{1}$
	d	$-1, \underline{1}$	$\underline{1}, -1$

The mixed strategy profile $(1/2 \cdot u + 1/2 \cdot d, 1/2 \cdot l + 1/2 \cdot r)$ is the only *mixed-strategy* Nash equilibrium.

Pure and Mixed Nash Equilibria

Row \ Column	l	r
u	$\underline{2}, \underline{1}$	$0, 0$
d	$0, 0$	$\underline{1}, \underline{2}$

(u, l), (d, r), and $(2/3 \cdot u + 1/3 \cdot d, 1/3 \cdot l + 2/3 \cdot r)$ are Nash equilibria.

Mixed Strategies

"We are reluctant to believe that our decisions are made at random. We prefer to be able to point to a reason for each action we take. Outside of Las Vegas we do not spin roulettes."

A. Rubinstein (1991). *Comments on the Interpretation of Game Theory*. Econometrica 59, pp. 909 - 924.

Mixed Strategies

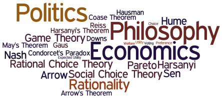
What does it mean to play a mixed strategy?

Mixed Strategies

What does it mean to play a mixed strategy?

- Mixed strategies are used to confuse your opponent (e.g., matching pennies games).
- A players mixed strategy is the belief of the *other* player about what that player will do.
- Mixed strategies are a concise description of what might happen in repeated play of the game.
- Mixed strategies describe population dynamics: After selecting 2 agents from a population, a mixed strategy is the probability of getting an agent who will play one pure strategy or another.

Nash Equilibria



Column

Row	l	r
u	$\underline{1}, -1$	$-1, \underline{1}$
d	$-1, \underline{1}$	$\underline{1}, -1$

- Some games may not have any pure strategy Nash equilibrium.

Nash Equilibria

Nash's Theorem. Any finite game has at least one mixed-strategy Nash equilibrium.

Nash Equilibria

	Column: l	Column: r
Row: u	$\underline{2}, \underline{1}$	$0, 0$
Row: d	$0, 0$	$\underline{1}, \underline{2}$

- There may be more than one Nash equilibria.

Nash Equilibria

Row \ Column	l	r
u	$\underline{2}, \underline{1}$	$0, 0$
d	$0, 0$	$\underline{1}, \underline{2}$

- Components of Nash equilibria are not interchangeable: If (x, y) and (x', y') are Nash equilibria in a 2-player game, then (x, y') and (x', y) may not be a Nash equilibrium.

  For example, (u, l) and (d, r) are Nash equilibria but (u, r) is **not** a Nash equilibrium.

Why *should* the players play their component of a Nash equilibrium?

Why play a Nash equilibrium?

Self-Enforcing Agreements: Nash equilibria are recommended by being the only strategy combinations on which the players could make self-enforcing agreements, i.e., agreements that each has reason to respect, even without external enforcement mechanisms.

M. Risse (2000). *What is rational about Nash equilibria?*. Synthese, 124:3, pp. 361 - 384.