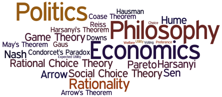
PHPE 400
Individual and Group Decision Making

Eric Pacuit
University of Maryland
pacuit.org

Important Games

Pareto



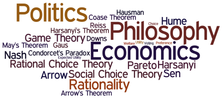
A strategy profile s **Pareto dominates** a strategy profile t provided *every* player strictly prefers the outcome in s than the outcome in t.

For example, with two players, a strategy profile (x, y) **Pareto dominates** a strategy profile (x', y') when

$$u_1(x, y) > u_1(x', y') \text{ and } u_2(x, y) > u_2(x', y').$$

A strategy profile s is **Pareto optimal** if there is no other profile that Pareto dominates s.

Pareto



Column

Row		a	b
	a	$3, 3$	$0, 0$
	b	$0, 0$	$1, 1$

- (a, a) Pareto dominates (a, b)
- (a, a) Pareto dominates (b, a), and
- (a, a) Pareto dominates (b, b).
- (a, a) is the only Pareto optimal profile.

Pareto

Column

Row	a	b
a	$2, 2$	$1, 3$
b	$3, 1$	$0, 0$

- (a, a) Pareto dominates (b, b)
- (a, b) Pareto dominates (b, b), and
- (b, a) Pareto dominates (b, b).
- (a, a), (a, b), and (b, a) are all Pareto optimal.

Coordination

		Column a	Column b
Row	a	$\underline{1}, \underline{1}$	$0, 0$
	b	$0, 0$	$\underline{1}, \underline{1}$

- Both (a, a) and (b, b) are Nash equilibria
- Both (a, a) and (b, b) are Pareto optimal
- The players want to coordinate by choosing the *same* action a or b.

Anti-Coordination

		Column	
		a	b
Row	a	$0, 0$	$\underline{1}, \underline{1}$
	b	$\underline{1}, \underline{1}$	$0, 0$

- Both (a, b) and (b, a) are Nash equilibria
- Both (a, b) and (b, a) are Pareto optimal
- The players want to mis-coordinate in which one player chooses a and the other chooses b.

Coordination and Competition

		Column	
		a	b
Row	a	$0, 0$	$\underline{2}, \underline{1}$
	b	$\underline{1}, \underline{2}$	$0, 0$

- Both (a, b) and (b, a) are Nash equilibria
- Both (a, b) and (b, a) are Pareto optimal
- Players want to mis-coordinate, and both prefer choosing b while the other chooses a.

Cailin O’Connor (2019). *The Origins of Unfairness: Social Categories and Cultural Evolution*. Oxford University Press.

Chicken

		Column a	Column b
Row	a	$2, 2$	$\underline{1}, \underline{3}$
Row	b	$\underline{3}, \underline{1}$	$0, 0$

- Both (a, b) and (b, a) are Nash equilibria
- All profiles except (b, b) are Pareto optimal
- Also called the "hawk-dove game"

Stag-Hunt

		Column	
		a	b
Row	a	$\underline{3}, \underline{3}$	$0, 2$
	b	$2, 0$	$\underline{1}, \underline{1}$

- (a, a) and (b, b) are the Nash equilibria
- (a, a) Pareto dominates (b, b)
- Choosing a may lead to a better outcome, but it is riskier.

Stag-Hunt

B. Skyrms (2004). *The Stag Hunt and the Evolution of Social Structure*. Cambridge University Press.

Prisoner's Dilemma

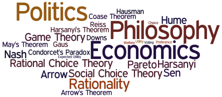
		Column	
		a	b
Row	a	$3, 3$	$0, \underline{4}$
	b	$\underline{4}, 0$	$\underline{1}, \underline{1}$

- (b, b) is the only Nash equilibrium
- (a, a) Pareto dominates (b, b)
- Typically, a is the "cooperate" action and b is the "defect" action.
- Often used to represent conflicts between individual rationality and cooperative behavior.

Prisoner's Dilemma

- Athletes using performance-enhancing drugs
- Two competing companies deciding advertising budgets
- Nation-states deciding to restrict CO2 emissions
- Two people meet and exchange closed bags, with the understanding that one of them contains money, and the other contains a purchase. Either player can choose to honor the deal by putting into his or her bag what he or she agreed, or he or she can defect by handing over an empty bag.
- `http://www.radiolab.org/story/golden-rule/`

Prisoner’s Dilemma



“Game theorists think it just plain wrong to claim that the Prisoners’ Dilemma embodies the essence of the problem of human cooperation. On the contrary, it represents a situation in which the dice are as loaded against the emergence of cooperation as they could possibly be. If the great game of life played by the human species were the Prisoner’s Dilemma, we wouldn’t have evolved as social animals!. . .. No paradox of rationality exists. Rational players don’t cooperate in the Prisoners’ Dilemma, because the conditions necessary for rational cooperation are absent in this game.” (Binmore, p. 63)

K. Binmore (2005). *Natural Justice*. Oxford University Press.

Prisoner’s Dilemma

- S. Kuhn, Prisoner’s Dilemma, Stanford Encyclopedia of Philosophy, plato.stanford.edu/entries/prisoner-dilemma/

- W. Poundstone, Prisoner’s Dilemma, Anchor, 1993

Issues

- There are games with a unique Nash equilibrium that is Pareto-dominated.
- There are games with multiple Nash equilibria.
- There are games that do not have any Nash equilibria.

Matching Pennies

		Column	
		l	r
Row	u	$1, -1$	$-1, 1$
	d	$-1, 1$	$1, -1$

Matching Pennies

Column

Row		l	r
	u	$\underline{1}, -1$	$-1, \underline{1}$
	d	$-1, \underline{1}$	$\underline{1}, -1$

There are no pure strategy Nash equilibria.

Mixed Strategies

A **mixed strategy** is a probability distribution over the set of pure strategies.

For instance, if a and b are the available actions, then the following are examples of mixed strategies:

- $1/2 \cdot a + 1/2 \cdot b$
- $1/3 \cdot a + 2/3 \cdot b$
- $4/5 \cdot a + 1/5 \cdot b$
- $\dots$

Matching Pennies

		Column	
		l	r
Row	u	$\underline{1}, -1$	$-1, \underline{1}$
	d	$-1, \underline{1}$	$\underline{1}, -1$

The mixed strategy profile $(1/2 \cdot u + 1/2 \cdot d, 1/2 \cdot l + 1/2 \cdot r)$ is the only *mixed-strategy* Nash equilibrium.

Pure and Mixed Nash Equilibria

		Column	
		l	r
Row	u	$\underline{2}, \underline{1}$	$0, 0$
	d	$0, 0$	$\underline{1}, \underline{2}$

(u, l), (d, r), and $(2/3 \cdot u + 1/3 \cdot d, 1/3 \cdot l + 2/3 \cdot r)$ are Nash equilibria.

Mixed Strategies

"We are reluctant to believe that our decisions are made at random. We prefer to be able to point to a reason for each action we take. Outside of Las Vegas we do not spin roulettes."

A. Rubinstein (1991). *Comments on the Interpretation of Game Theory*. Econometrica 59, pp. 909 - 924.

Mixed Strategies

What does it mean to play a mixed strategy?

Mixed Strategies

What does it mean to play a mixed strategy?

- Mixed strategies are used to confuse your opponent (e.g., matching pennies games).
- A players mixed strategy is the belief of the *other* player about what that player will do.
- Mixed strategies are a concise description of what might happen in repeated play of the game.
- Mixed strategies describe population dynamics: After selecting 2 agents from a population, a mixed strategy is the probability of getting an agent who will play one pure strategy or another.

Nash Equilibria

Nash's Theorem. Any finite game has at least one mixed-strategy Nash equilibrium.

Why *should* the players play their component of a Nash equilibrium?

Why play a Nash equilibrium?

Self-Enforcing Agreements: Nash equilibria are recommended by being the only strategy combinations on which the players could make self-enforcing agreements, i.e., agreements that each has reason to respect, even without external enforcement mechanisms.

M. Risse (2000). *What is rational about Nash equilibria?*. Synthese, 124:3, pp. 361 - 384.

Row	Column	l	c	r
	u	$4, 6$	$5, 4$	$0, 0$
	m	$5, 7$	$4, 8$	$0, 0$
	d	$0, 0$	$0, 0$	$1, 1$

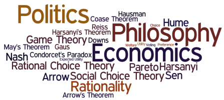
		Column		
		l	c	r
Row	u	$4, \underline{6}$	$\underline{5}, 4$	$0, 0$
	m	$\underline{5}, 7$	$4, \underline{8}$	$0, 0$
	d	$0, 0$	$0, 0$	$\underline{1}, \underline{1}$

(d, r) is a Nash equilibrium, but it is **not self-enforcing**

		Column	
		l	r
Row	u	$0, 0$	$4, 2$
	d	$2, 4$	$3, 3$

		Column	
		l	r
Row	u	$0, 0$	$\underline{4}, \underline{2}$
	d	$\underline{2}, \underline{4}$	$3, 3$

(d, r) is **not** a Nash equilibrium, but it is **self-enforcing**



Self-Enforcing Agreements: Nash equilibria are recommended by being the only strategy combinations on which the players could make self-enforcing agreements, i.e., agreements that each has reason to respect, even without external enforcement mechanisms.

- There are Nash equilibria that are not self-enforcing
- There are self-enforcing outcomes that are not Nash equilibria

Is a Nash equilibrium *guaranteed* by players that are rational and have *common knowledge* of each others' rationality?

Column

Row	l	c	r
u	$3, 2$	$0, 0$	$2, 3$
m	$0, 0$	$1, 1$	$0, 0$
d	$2, 3$	$0, 0$	$3, 2$

Column

Row	l	c	r
u	$\underline{3}, 2$	$0, 0$	$2, \underline{3}$
m	$0, 0$	$\underline{1}, \underline{1}$	$0, 0$
d	$2, \underline{3}$	$0, 0$	$\underline{3}, 2$

(m, c) is the unique Nash equilibrium

Column

Row	l	c	r
u	$\underline{3}, 2$	$0, 0$	$2, \underline{3}$
m	$0, 0$	$\underline{1}, \underline{1}$	$0, 0$
d	$2, \underline{3}$	$0, 0$	$\underline{3}, 2$

u, d, l, and r are all **rationalizable**

		Column		
		l	c	r
Row	u	$\underline{3}, 2$	$0, 0$	$2, \underline{3}$
	m	$0, 0$	$\underline{1}, \underline{1}$	$0, 0$
	d	$2, \underline{3}$	$0, 0$	$\underline{3}, 2$

Row plays d because she thought Column would play r

Column

Row	l	c	r
u	$\underline{3}, 2$	$0, 0$	$2, \underline{3}$
m	$0, 0$	$\underline{1}, \underline{1}$	$0, 0$
d	$2, \underline{3}$	$0, 0$	$\underline{3}, 2$

Column plays l because she thought Row would play d

Column

Row	l	c	r
u	$\underline{3}, 2$	$0, 0$	$2, \underline{3}$
m	$0, 0$	$\underline{1}, \underline{1}$	$0, 0$
d	$\boxed{2, \underline{3}}$	$0, 0$	$\underline{3}, 2$

Column was correct, but Row was wrong.
Both players are **rational**.

		Column			
		l	c	r	x
Row	u	$\underline{3}, 2$	$0, 0$	$2, \underline{3}$	$0, -5$
	m	$0, 0$	$\underline{1}, \underline{1}$	$0, 0$	$\underline{100}, -5$
	d	$2, \underline{3}$	$0, 0$	$\underline{3}, 2$	$1, -3$

Not every strategy is rationalizable

Column

Row	l	c	r	x
u	$\underline{3}, 2$	$0, 0$	$2, \underline{3}$	$0, -5$
m	$0, 0$	$\underline{1}, \underline{1}$	$0, 0$	$\underline{100}, -5$
d	$2, \underline{3}$	$0, 0$	$\underline{3}, 2$	$1, -3$

Not every strategy is rationalizable:
Row can't play m *because* she thinks Column will play x

An action a **strictly dominates** another action b for player i when i's utility is strictly better choosing a than choosing b no matter what actions are chosen by the other players.

Example

Row		Column: l	Column: r
	u	$5, 5$	$-100, 4$
	d	$0, 1$	$0, 0$

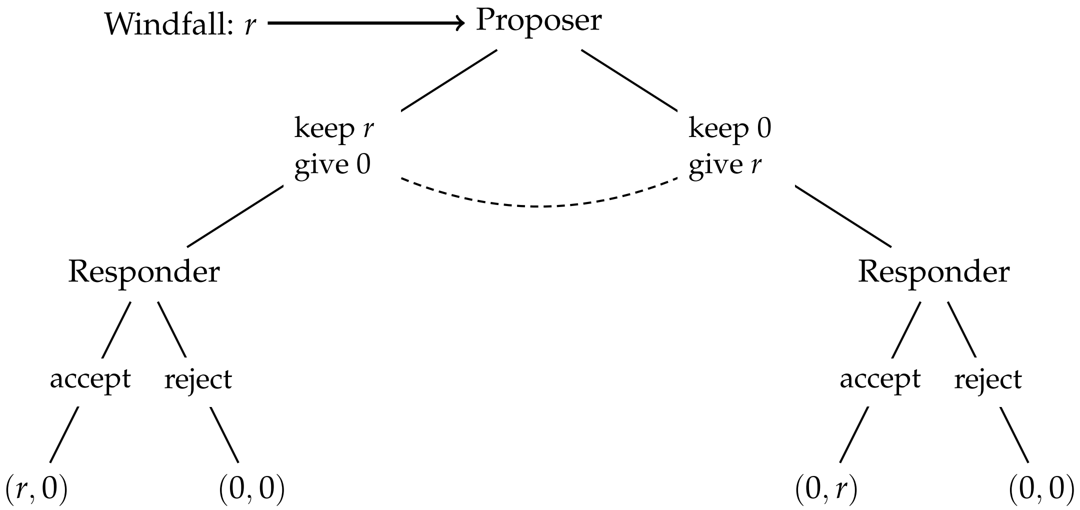
Example

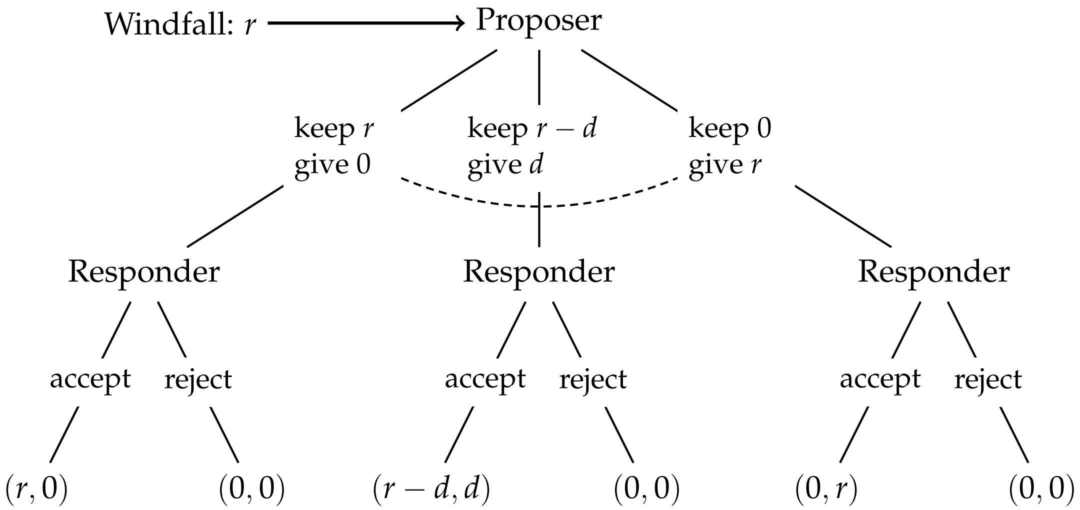
		Column	
		l	r
Row	u	$5, 5$	$-100, 4$
	d	$0, 1$	$0, 0$

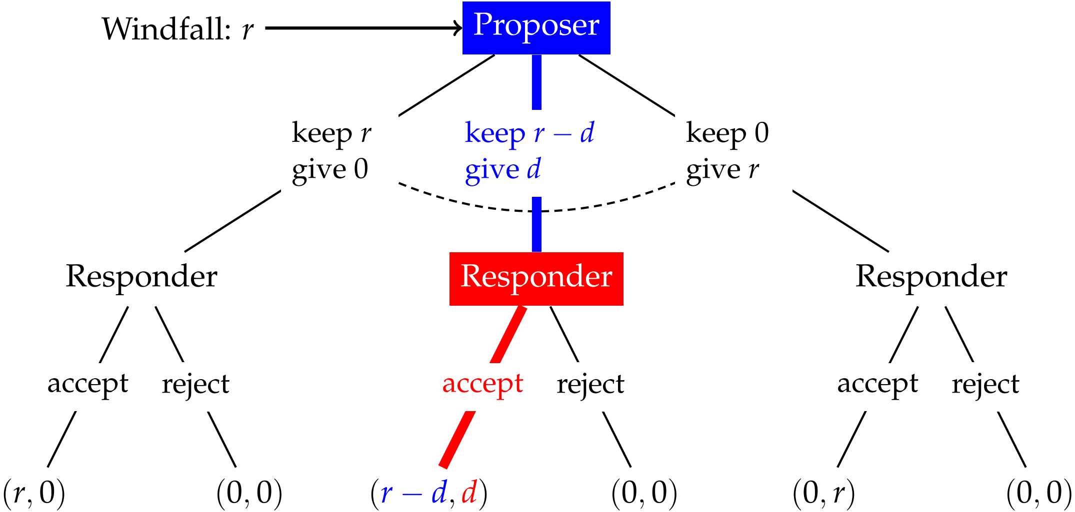
Since r is **strictly dominated** by l, Column will not play r.
Then, the best response for Row is u.

Ultimatum Game

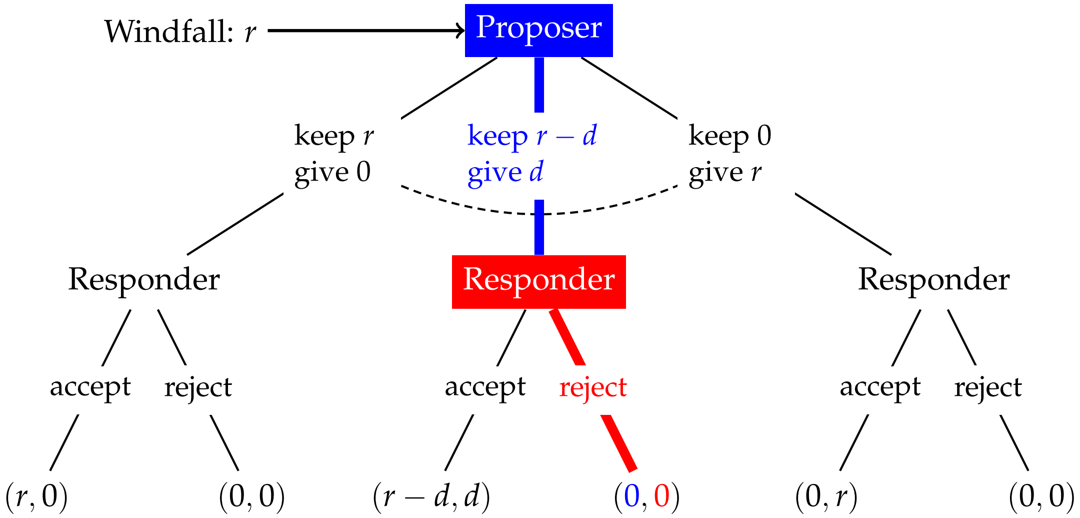
Ultimatum Game: Two players receive a windfall. One of the players suggests a division. After learning of the first player's proposal, the second must either accept or reject it. If the second accepts, both receive the amounts suggested by the first, otherwise they receive nothing.


Windfall: r
Proposer
keep r give 0
keep 0 give r
Responder
Responder
accept
reject
accept
reject
$(r, 0)$
$(0, 0)$
$(0, r)$
$(0, 0)$


Windfall: r
Proposer
keep r
give 0
keep $r-d$
give d
keep 0
give r
Responder
Responder
Responder
accept
reject
accept
reject
accept
reject
$(r, 0)$
$(0, 0)$
$(r-d, d)$
$(0, 0)$
$(0, r)$
$(0, 0)$



Proposer gets $r - d$ and Responder gets d



Proposer gets 0 and Responder gets 0

Sequential Rationality

If the proposer offers a split which gives the second any positive amount, the second does strictly worse by refusing the offer. So, no rejection strategies are sequentially rational.

Knowing this, the first player ought to offer the smallest amount possible to the second player.

This is not what is observed:

> ...offers typically average about 30-40 percent of the total, with a 50-50 split often the mode. Offers of less than 20 percent are frequently rejected. These facts are not now in question. What remains controversial is how to interpret the facts and how best to incorporate what we have learned into a more descriptive version of game theory.
>
> (p. 210, Camerer and Thaler)

C. Camerer and R. Thaler (1995). *Anomalies: Ultimatums, Dictators and Manners*. The Journal of Economic Perspectives, 9(2), pp. 209-219.

- Rejecting low offers is impossible to reconcile with a theory of *payoff maximization*.

- Rejecting low offers is impossible to reconcile with a theory of *payoff maximization*.

- Making a non-zero offer is consistent with payoff maximization, if a proposer believes that the responder will reject too low an offer.

- Rejecting low offers is impossible to reconcile with a theory of *payoff maximization*.

- Making a non-zero offer is consistent with payoff maximization, if a proposer believes that the responder will reject too low an offer.
    - However, offers are typically larger than the amount that proposers believe would result in acceptance.

    Joseph Henrich, Robert Boyd, Samuel Bowles, Colin Camerer, Ernst Fehr, Herbert Gintis, and Richard McElreath (2001). *In search of homo economicus: Behavioral experiments in 15 small-scale societies*. American Economic Review, 91(2), pp. 73–78.

Dictator Game

In the dictator game, the first player, called the Allocator, makes a unilateral decision regarding the split of the pie. The second player, the Recipient, has no choice and receives only the amount that the dictator decides to give.

Since dictators have no monetary incentives to give, a payoff-maximizing dictator would keep the whole amount.

Dictator Game

Experimental Regularity: A significant number of Allocators give some money in the dictator game. Moreover, the distribution of donations tend to be bimodal, with peaks at zero and at half the total.

Daniel Kahneman, Jack L. Knetsch, and Richard Thaler (1986). *Fairness as a Constraint on Profit Seeking: Entitlements in the Market*. American Economic Review, 76, pp. 728 - 741.

Christoph Engel (2011). *Dictator games: A meta study*. Experimental Economics, 14(4), pp. 583 - 610.

Methodological Individualism

Traditional economic models presume that individuals do not take an interest in the interests of those with whom they interact. **More particularly, the assumption of *non-tuism* implies that the utility function of each individual, as a measure of her preferences, is strictly independent of the utility functions of those with whom she interacts.**

Methodological Individualism

Traditional economic models presume that individuals do not take an interest in the interests of those with whom they interact. **More particularly, the assumption of *non-tuism* implies that the utility function of each individual, as a measure of her preferences, is strictly independent of the utility functions of those with whom she interacts.** ... Interestingly, this idea is quite different from the usual egoistic assumption: a non-tuist may be a caring, altruistic human being, but when involved in an economic exchange, she must necessarily regard her own interest as paramount.

Methodological Individualism

Traditional economic models presume that individuals do not take an interest in the interests of those with whom they interact. **More particularly, the assumption of *non-tuism* implies that the utility function of each individual, as a measure of her preferences, is strictly independent of the utility functions of those with whom she interacts.** ... Interestingly, this idea is quite different from the usual egoistic assumption: a non-tuist may be a caring, altruistic human being, but when involved in an economic exchange, she must necessarily regard her own interest as paramount. Thus non-tuism is important insofar as it defines the scope of economic activities. When tuism to some degree motivates one's conduct, then it ceases to be wholly economic.

Cristina Bicchieri and Jiji Zhang (2012). *An Embarrassment of Riches: Modeling Social Preferences in Ultimatum Games*. Handbook of the Philosophy of Science, Volume 13: Philosophy of Economics.