PHPE 400
Individual and Group Decision Making

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Just Enough Game Theory

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- the "structure" of the decision problem (what information do the players have?, what order do they act in?, how many times do they repeat their interaction?, etc.)

*It does **not** specify the actions that the players **do take**.*

Simultaneous-move

In **simultaneous-move games**, also called **strategic games** or **normal form games**, all players select an action simultaneously, without knowing what the others will do (though they can certainly *reason* about what the other players are expected to do).

Strategic Games

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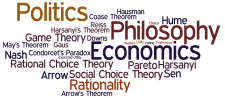
Strategic Games



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- N is a finite set of **players**
- for each $i \in N$, A_i is a nonempty set of **actions** (also called **strategies**)
- for each $i \in N$, u_i is a **utility function** for player i on the set of outcomes (called strategy profiles): $u_i : \times_{k \in N} A_k \to \mathbb{R}$.

Strategic Games: Example

Row \ Column	l	r
u	$2, 1$	$0, 0$
d	$0, 0$	$1, 2$

- $N = \{Row, Column\}$
- $A_{Row} = \{u, d\}$, $A_{Column} = \{l, r\}$
- $u_{Row} : A_{Row} \times A_{Column} \to \mathbb{R}$, $u_{Column} : A_{Row} \times A_{Column} \to \mathbb{R}$ with $u_{Row}(u, l) = u_{Column}(d, r) = 2$, $u_{Row}(d, r) = u_{Column}(u, l) = 1$, and $u_{Row}(d, l) = u_{Column}(d, l) = u_{Row}(u, r) = u_{Column}(u, r) = 0$.

Strategy Profiles

Row \ Column	l	r
u	$2, 1$	$0, 0$
d	$0, 0$	$1, 2$

A **strategy profile** is a list of actions, one for each player, that represents the outcome of the game.

The 4 possible strategy profiles in the above game are

$$\{(u, l), (d, l), (u, r), (d, r)\}$$

Important Point

The goal of the players is to maximize **their own utility**.

The players' utilities represent all of their opinions about the outcome of the game (e.g., "winning the game" or "beating the other player").

Solution Concept

A **solution concept** is a systematic description of the outcomes (i.e., the strategy profiles) that may emerge in a family of games.

This is the starting point for most of game theory and includes many variants.

These are usually thought of as the embodiment of “rational behavior” in some way and used to analyze game situations.

Best Response

The **best response** for player i to a list of the other players' actions is the action that maximizes i's utility *assuming that the other players choose their action in the list*.

Best Response

Row \ Column	l	r
u	$2, 1$	$0, 0$
d	$0, 0$	$1, 2$

Row: The best response to l is u and the best response to r is d

Best Response

Row \ Column	l	r
u	$\underline{2}, \underline{1}$	$0, 0$
d	$0, 0$	$\underline{1}, \underline{2}$

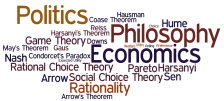
Row: The best response to l is u and the best response to r is d

Column: The best response to u is l and the best response to d is r

Nash Equilibrium

A strategy profile is a **Nash equilibrium** if every player's strategy is a best response to the other player's strategies.

Nash Equilibrium: Example



Row \ Column	l	r
u	$\underline{2}, \underline{1}$	$0, 0$
d	$0, 0$	$\underline{1}, \underline{2}$

(u, l) is a Nash Equilibrium

(d, r) is a Nash Equilibrium

Nash Equilibrium: Example

Row \ Column	l	r
u	$\underline{2}, \underline{1}$	$0, 0$
d	$0, 0$	$\underline{1}, \underline{2}$

(u, l) is a Nash Equilibrium

(u, r) is **not** a Nash Equilibrium

(d, r) is a Nash Equilibrium

(d, l) is **not** a Nash Equilibrium

Nash Equilibria

Row \ Column	l	r
u	$1, 1$	$2, 1$
d	$1, 2$	$3, 3$

Nash Equilibria
(u, l) and (d, r)

Nash Equilibria

Row \ Column	l	r
u	$1, 1$	$2, 1$
d	$1, 2$	$3, 3$

Nash Equilibria
(u, l) and (d, r)

Row \ Column	l	r
u	$2, 2$	$2, 1$
d	$1, 3$	$10, 2$

Nash Equilibria
(u, l)

Nash Equilibria

Row \ Column	l	r
u	$1, 1$	$2, 1$
d	$1, 2$	$3, 3$

Nash Equilibria
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Row \ Column	l	r
u	$2, 2$	$2, 1$
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Nash Equilibria
(u, l)

Row \ Column	l	r
u	$0, 1$	$2, 1$
d	$2, 0$	$2, 2$

Nash Equilibria
(u, r) and (d, r)