PHPE 400
Individual and Group Decision Making

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	Rain (s_1)	No rain (s_2)
Take umbrella (A)	encumbered, dry (o_1)	encumbered, dry (o_1)
Leave umbrella (B)	free, wet (o_2)	free, dry (o_3)

$$A(s_1) = A(s_2) = o_1$$

$$B(s_1) = o_2, B(s_2) = o_3$$

	Rain (s_1)	No rain (s_2)
Take umbrella (A)	encumbered, dry (o_1)	encumbered, dry (o_1)
Leave umbrella (B)	free, wet (o_2)	free, dry (o_3)

Suppose that $P(s_1) = 0.6$ and $P(s_2) = 0.4$
(the decision maker believes that there is a 60% chance that it will rain).

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Take umbrella (A)	encumbered, dry (o_1)	encumbered, dry (o_1)
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Suppose that $P(s_1) = 0.6$ and $P(s_2) = 0.4$
(the decision maker believes that there is a 60% chance that it will rain).

Suppose that the decision maker's utility for the outcomes is:
$u(o_1) = 5$, $u(o_2) = 0$ and $u(o_3) = 10$.

	Rain (s_1) $P(s_1) = 0.6$	No rain (s_2) $P(s_2) = 0.4$
Take umbrella (A)	encumbered, dry (o_1) $u(o_1) = 5$	encumbered, dry (o_1) $u(o_1) = 5$
Leave umbrella (B)	free, wet (o_2) $u(o_2) = 0$	free, dry (o_3) $u(o_3) = 10$

$$
\begin{aligned}
EU(A, u) &= P(s_1) \times u(A(s_1)) + P(s_2) \times u(A(s_2)) \\
EU(B, u) &= P(s_1) \times u(B(s_1)) + P(s_2) \times u(B(s_2))
\end{aligned}
$$

	Rain (s_1) $P(s_1) = 0.6$	No rain (s_2) $P(s_2) = 0.4$
Take umbrella (A)	encumbered, dry (o_1) $u(o_1) = 5$	encumbered, dry (o_1) $u(o_1) = 5$
Leave umbrella (B)	free, wet (o_2) $u(o_2) = 0$	free, dry (o_3) $u(o_3) = 10$

$$\begin{aligned} EU(A,u) &= 0.6 \times 5 + 0.4 \times 5 &= 5 \\ EU(B,u) &= 0.6 \times 0 + 0.4 \times 10 &= 4 \end{aligned}$$

$EU(A,u) > EU(B,u)$, so the decision maker strictly prefers A to B.

	Rain (s_1) $P(s_1) = 0.6$	No rain (s_2) $P(s_2) = 0.4$
Take umbrella (A)	encumbered, dry (o_1) $u'(o_1) = 4$	encumbered, dry (o_1) $u'(o_1) = 4$
Leave umbrella (B)	free, wet (o_2) $u'(o_2) = 2$	free, dry (o_3) $u'(o_3) = 8$

$$\begin{aligned} EU(A, u') &= 0.6 \times 4 + 0.4 \times 4 &= 4 \\ EU(B, u') &= 0.6 \times 2 + 0.4 \times 8 &= 4.4 \end{aligned}$$

$EU(A, u') < EU(B, u')$, so the decision maker strictly prefers B to A.

	Rain (s_1) $P(s_1) = 0.6$	No rain (s_2) $P(s_2) = 0.4$
Take umbrella (A)	encumbered, dry (o_1)	encumbered, dry (o_1)
Leave umbrella (B)	free, wet (o_2)	free, dry (o_3)

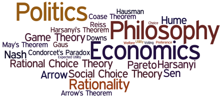
$$u(o_3) = 10 > u(o_1) = 5 > u(o_2) = 0$$
$$EU(A, u) = 0.6 \times 5 + 0.4 \times 5 = 5 > EU(B, u) = 0.6 \times 0 + 0.4 \times 10 = 4$$

$$u'(o_3) = 8 > u'(o_1) = 4 > u'(o_2) = 2$$
$$EU(A, u') = 0.6 \times 4 + 0.4 \times 4 = 4 < EU(B, u') = 0.6 \times 2 + 0.4 \times 8 = 1.2 + 3.2 = 4.4$$

For all acts A and B and utility functions u,

if $EU(A, u) > EU(B, u)$ and u' is a linear transformation of u (i.e., $u'(\cdot) = a \times u(\cdot) + b$ for some $a, b \in \mathbb{R}$ with $a > 0$), then $EU(A, u') > EU(B, u')$

Strict Dominance



	s_1	s_2	s_3
A	2	3	1
B	1	2	0
C	1	4	0

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers B to A?

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers C to A?

Strict Dominance

	s_1	s_2	s_3
A	2	3	1
B	1	2	0
C	1	4	0

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers B to A? No!

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers C to A? Yes!

Strict Dominance

	s_1	s_2	s_3
A	2	3	1
B	1	2	0
C	1	4	0

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers B to A? No!

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers C to A? Yes!

X **strictly dominates** Y when for all states s, $u(X(s)) > u(Y(s))$.

- A strictly dominates B

Strict Dominance

	s_1	s_2	s_3
A	2	3	1
B	1	2	0
C	1	4	0

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers B to A? No!

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers C to A? Yes!

X **strictly dominates** Y when for all states s, $u(X(s)) > u(Y(s))$.

- A strictly dominates B
- A does not strictly dominate C

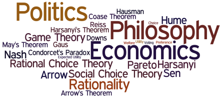
Weak Dominance

	s_1	s_2	s_3
A	2	3	1
B	1	2	1
C	2	3	1

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers B to A?

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers C to A?

Weak Dominance

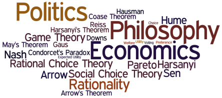


	s_1	s_2	s_3
A	2	3	1
B	1	2	1
C	2	3	1

Does the decision maker strictly prefer A to B?

Does the decision maker strictly prefer A to C?

Weak Dominance



	s_1	s_2	s_3
A	2	3	1
B	1	2	1
C	2	3	1

Does the decision maker strictly prefer A to B? Depends...

Does the decision maker strictly prefer A to C? No!

X **weakly dominates** Y when for all states s, $u(X(s)) \geq u(Y(s))$ and there is some s' such that $u(X(s')) > u(Y(s'))$.

- A weakly dominates B

Weak Dominance

	s_1	s_2	s_3
A	2	3	1
B	1	2	1
C	2	3	1

Does the decision maker strictly prefer A to B? Depends...

Does the decision maker strictly prefer A to C? No!

X **weakly dominates** Y when for all states s, $u(X(s)) \geq u(Y(s))$ and there is some s' such that $u(X(s')) > u(Y(s'))$.

- A weakly dominates B
- A does not weakly dominate C

R. Nozick. *Newcomb's Problem and Two Principles of Choice*. 1969.

There are two boxes in front of us:

- box A, which contains \$1,000;
- box B, which contains either \$1,000,000 or nothing.

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We have two choices:

- we open only box B.
- we open both box A and box B;

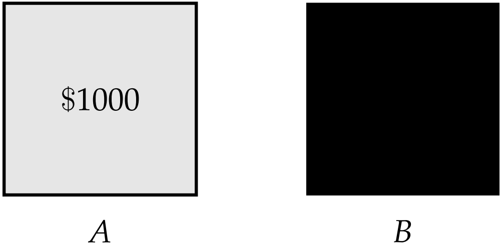
There are two boxes in front of us:

- box A, which contains \$1,000;
- box B, which contains either \$1,000,000 or nothing.

We have two choices:

- we open only box B.
- we open both box A and box B;

You can see inside box A, but not inside box B. We can keep whatever is inside any box we open, but we may not keep what is inside a box that we do not open.



Choice:

- one-box: choose box *B*
- two-box: choose box *A* and *B*

A famous example: Newcomb's paradox

A very powerful being, who has been invariably accurate in his predictions about our behavior in the past, has already acted in the following way:

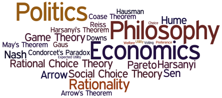
1. If he has predicted we will open just box B, he has put \$1,000,000 in box B.
2. If he has predicted we open both boxes, he has put nothing in box B.

What should we do?

Newcomb's paradox

you		*pred_B*	*pred_AB*
	B	$1M$	0
	AB	$1M+1T$	$1T$

Newcomb's paradox



you		*pred_B*	*pred_AB*
	B	$1M$	0
	AB	$1M{+}1T$	$1T$

Principle of dominance: take both boxes.

Newcomb's paradox

- $P(pred\_B \mid B)$: The probability that the wizard predicted you would choose box B *given that you decided to choose box* B.
- $P(pred\_AB \mid B)$: The probability that the wizard predicted you would choose both boxes *given that you decided to choose box* B.

Newcomb's paradox

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Newcomb's paradox

- ✓ $P(pred\_B \mid B)$: The probability that the wizard predicted you would choose box B *given that you decided to choose box* B.
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- ✗ $P(pred\_B \mid AB)$: The probability that the wizard predicted you would choose box B *given that you decided to choose both boxes*.
- ✓ $P(pred\_AB \mid AB)$: The probability that the wizard predicted you would choose both boxes *given that you decided to choose both boxes*.

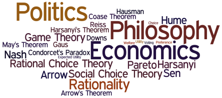
Newcomb's paradox

you	$pred\_B$	$pred\_AB$
B	$1M$	0
AB	$1M$+$1T$	$1T$

Expected utility maximization: take box B.

$$P(pred\_B \mid B)1M + P(pred\_AB \mid B)0 > P(pred\_B \mid AB)(1M + 1T) + P(pred\_AB \mid AB)1T$$

Newcomb's Paradox



What the Predictor did yesterday is *probabilistically dependent* on the choice today, but *causally independent* of today's choice.

Act-state independence: For all states s and actions X, $P(s) = P(s \mid X)$

J. Collins. *Newcomb's Problem*. International Encyclopedia of Social and Behavioral Sciences, 1999.