

PHPE 400

Individual and Group Decision Making

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Summary



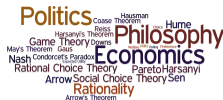
- ▶ For a *cardinal* utility function (measured on an interval scale) that represents a decision maker's preferences over outcomes, the decision maker compares lotteries using **expected utility** based on this utility function.

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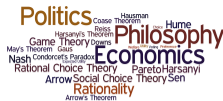
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 - ▶ The utility function representing the decision maker's preferences is unique up to *linear transformations*.

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 - ▶ The utility function representing the decision maker's preferences is unique up to *linear transformations*.
- ▶ **Rational preferences** over lotteries are characterized by satisfying Transitivity, Completeness, and the Independence Axiom.

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 - ▶ The utility function representing the decision maker's preferences is unique up to *linear transformations*.
- ▶ **Rational preferences** over lotteries are characterized by satisfying Transitivity, Completeness, and the Independence Axiom.
- ▶ If a decision maker violates the Independence Axiom, then no utility function can be used to rank lotteries using expected utility.

Allais Paradox



	Red (1)	White (89)	Blue (10)
L_3	1M	0	1M
L_4	0	0	5M

Allais Paradox



	Red (1)	White (89)	Blue (10)
L_1	1M	1M	1M
L_2	0	1M	5M
L_3	1M	0	1M
L_4	0	0	5M

Independence and Allais

$$\left(\frac{1}{100} \cdot 1M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 1M\right) \quad P \quad \left(\frac{1}{100} \cdot 0M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 5M\right)$$



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iff

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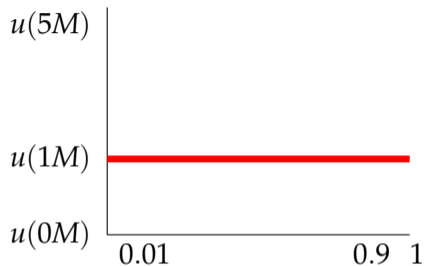
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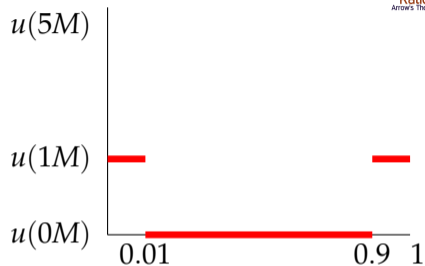
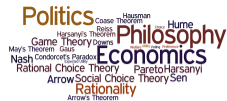
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Allais Paradox

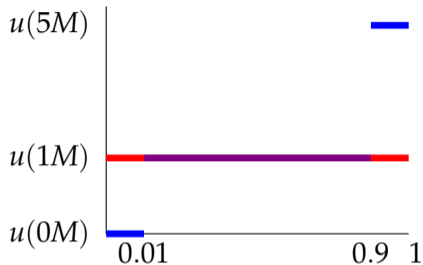


$$L_1 = 0.01 \cdot 1M + 0.89 \cdot 1M + 0.1 \cdot 1M$$



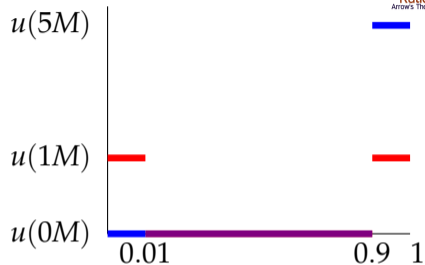
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Allais Paradox



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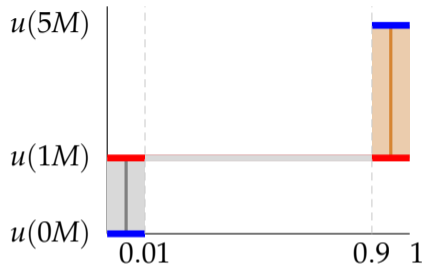


$$L_3 = 0.01 \cdot 1M + 0.89 \cdot 0M + 0.1 \cdot 1M$$

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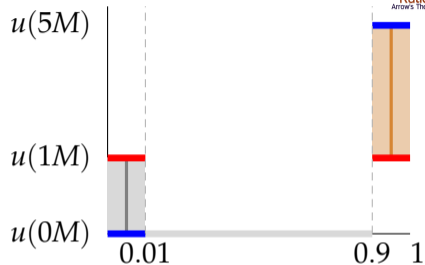
Politics
Coase
Theorem
Hausman
Hume
Philosophy
Harsanyi's Theorem
Rais
Game Theory
Downs
Nash
May's Theorem
Condorcet's Paradox
Gaus
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Allais Paradox



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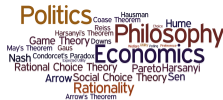
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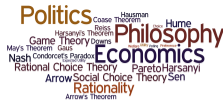
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(b) those who choose L_1 and L_4 are irrational.

Allais Paradox



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(a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or

(b) those who choose L_1 and L_4 are irrational.

Rather, people's utility functions (*their rankings over outcomes*) are often far more complicated than the monetary bets would indicate....

L. Buchak. *Risk and Rationality*. Oxford University Press, 2013.