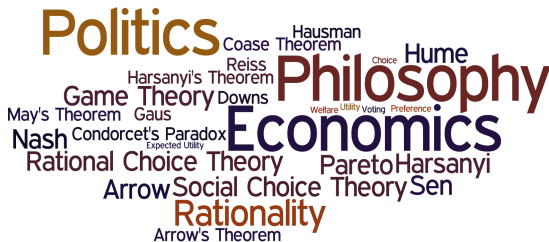


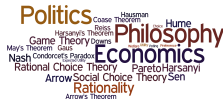
PHPE 400

Individual and Group Decision Making

Eric Pacuit
University of Maryland
pacuit.org



Independence

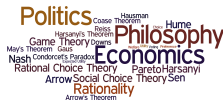


For all $L_1, L_2, L_3 \in \mathcal{L}$ and $0 < p \leq 1$,

$L_1 P L_2$ if, and only if, $(p \cdot L_1 + (1 - p) \cdot L_3) P (p \cdot L_2 + (1 - p) \cdot L_3)$.

$L_1 I L_2$ if, and only if, $(p \cdot L_1 + (1 - p) \cdot L_3) I (p \cdot L_2 + (1 - p) \cdot L_3)$.

Summary



- For a *cardinal* utility function (measured on an interval scale) that represents a decision maker's preferences over outcomes, the decision maker compares lotteries using **expected utility** based on this utility function.

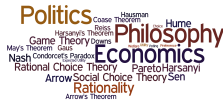
A word cloud featuring names of economists and political theorists, and their associated theories. The words are arranged in a circular pattern. The most prominent words are 'Economics' (large, dark blue), 'Philosophy' (large, dark red), 'Politics' (large, orange), and 'Rationality' (medium, orange). Other visible words include 'Hume', 'Harsanyi', 'Pareto', 'Arrow', 'Nash', 'Game Theory', 'Social Choice Theory', 'Rational Choice Theory', 'Coase Theorem', 'Hausman', 'Reiss', 'Downs', 'May's Theorem', 'Gaus', 'Condorcet's Paradox', 'Arrow's Theorem', 'Sen', 'Theory', 'Pareto', 'Harsanyi', 'Arrow's Social Choice Theory', 'Rationality', 'Arrow's Theorem', 'Rational Choice Theory', 'Nash', 'Condorcet's Paradox', 'Gaus', 'May's Theorem', 'Game Theory', 'Downs', 'Harsanyi's', 'Reiss', 'Coase Theorem', 'Hausman', 'Hume'.

- 3 / 10

- 3 / 10

Evaluating rational choice axioms

Allais Paradox



	Red (0.01)	White (0.89)	Blue (0.10)
L_1	1M	1M	1M
L_2	0	1M	5M

A word cloud featuring names of economists and political theorists, and names of theories. The words are arranged in a circular pattern. The most prominent words are 'Economics' (large, dark blue), 'Philosophy' (large, dark blue), 'Politics' (large, orange), 'Rationality' (medium, orange), 'Rational Choice Theory' (medium, dark blue), 'Arrow's Theorem' (medium, dark blue), 'Game Theory' (medium, dark blue), 'Harsanyi's Theorem' (medium, dark blue), 'Nash' (medium, dark blue), 'Pareto' (medium, dark blue), 'Arrow' (medium, dark blue), 'Social Choice' (medium, dark blue), 'Theory Sen' (medium, dark blue), 'Hume' (medium, dark blue), 'Hausman' (medium, dark blue), 'Reiss' (medium, dark blue), 'Coase Theorem' (medium, dark blue), 'May's Theorem' (medium, dark blue), 'Gaus' (medium, dark blue), 'Condorcet's Paradox' (medium, dark blue), 'Rationality' (medium, orange), 'Arrow's Theorem' (medium, dark blue), 'Game Theory' (medium, dark blue), 'Harsanyi's Theorem' (medium, dark blue), 'Nash' (medium, dark blue), 'Pareto' (medium, dark blue), 'Arrow' (medium, dark blue), 'Social Choice' (medium, dark blue), 'Theory Sen' (medium, dark blue), 'Hume' (medium, dark blue), 'Hausman' (medium, dark blue), 'Reiss' (medium, dark blue), 'Coase Theorem' (medium, dark blue), 'May's Theorem' (medium, dark blue), 'Gaus' (medium, dark blue), 'Condorcet's Paradox' (medium, dark blue).

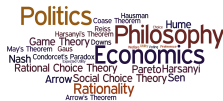
5 / 10

A word cloud featuring names of economists and political theorists, and names of theories. The words are arranged in a circular pattern. The most prominent words are 'Economics', 'Philosophy', 'Politics', 'Rationality', 'Game Theory', 'Arrow', 'Social Choice Theory', 'Pareto', 'Harsanyi', 'Nash', 'Condorcet's Paradox', 'May's Theorem', 'Gaus', 'Hausman', 'Reiss', 'Coase Theorem', 'Hume', 'Rational Choice Theory', 'Arrow's Theorem', and 'Sen'. The words are in various colors (blue, green, yellow, red, black) and sizes, with 'Economics' and 'Philosophy' being the largest.

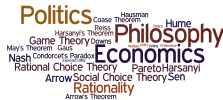
5 / 10

Independence and Allais

$$\left(\frac{1}{100} \cdot 1M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 1M\right) \quad P \quad \left(\frac{1}{100} \cdot 0M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 5M\right)$$



Independence and Allais



$$\left(\frac{1}{100} \cdot 1M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 1M\right) \quad P \quad \left(\frac{1}{100} \cdot 0M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 5M\right)$$

iff

$$\frac{11}{100} \cdot \left(\frac{1}{11} \cdot 1M + \frac{10}{11} \cdot 1M\right) + \frac{89}{100} \cdot 1M \quad P \quad \frac{11}{100} \cdot \left(\frac{1}{11} \cdot 0M + \frac{10}{11} \cdot 5M\right) + \frac{89}{100} \cdot 1M$$

Independence and Allais



$$\left(\frac{1}{100} \cdot 1M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 1M\right) \quad P \quad \left(\frac{1}{100} \cdot 0M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 5M\right)$$

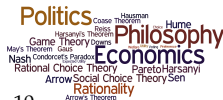
iff

$$\frac{11}{100} \cdot \left(\frac{1}{11} \cdot 1M + \frac{10}{11} \cdot 1M\right) + \frac{89}{100} \cdot 1M \quad P \quad \frac{11}{100} \cdot \left(\frac{1}{11} \cdot 0M + \frac{10}{11} \cdot 5M\right) + \frac{89}{100} \cdot 1M$$

iff

$$\left(\frac{1}{11} \cdot 1M + \frac{10}{11} \cdot 1M\right) \quad P \quad \left(\frac{1}{11} \cdot 0M + \frac{10}{11} \cdot 5M\right)$$

Independence and Allais



$$\left(\frac{1}{100} \cdot 1M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 1M\right) \quad P \quad \left(\frac{1}{100} \cdot 0M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 5M\right)$$

iff

$$\frac{11}{100} \cdot \left(\frac{1}{11} \cdot 1M + \frac{10}{11} \cdot 1M\right) + \frac{89}{100} \cdot 1M \quad P \quad \frac{11}{100} \cdot \left(\frac{1}{11} \cdot 0M + \frac{10}{11} \cdot 5M\right) + \frac{89}{100} \cdot 1M$$

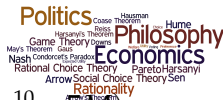
iff

$$\left(\frac{1}{11} \cdot 1M + \frac{10}{11} \cdot 1M\right) \quad P \quad \left(\frac{1}{11} \cdot 0M + \frac{10}{11} \cdot 5M\right)$$

iff

$$\frac{11}{100} \cdot \left(\frac{1}{11} \cdot 1M + \frac{10}{11} \cdot 1M\right) + \frac{89}{100} \cdot 0M \quad P \quad \frac{11}{100} \cdot \left(\frac{1}{11} \cdot 0M + \frac{10}{11} \cdot 5M\right) + \frac{89}{100} \cdot 0M$$

Independence and Allais



$$\left(\frac{1}{100} \cdot 1M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 1M\right) \quad P \quad \left(\frac{1}{100} \cdot 0M + \frac{89}{100} \cdot 1M + \frac{10}{100} \cdot 5M\right)$$

iff

$$\frac{11}{100} \cdot \left(\frac{1}{11} \cdot 1M + \frac{10}{11} \cdot 1M\right) + \frac{89}{100} \cdot 1M \quad P \quad \frac{11}{100} \cdot \left(\frac{1}{11} \cdot 0M + \frac{10}{11} \cdot 5M\right) + \frac{89}{100} \cdot 1M$$

iff

$$\left(\frac{1}{11} \cdot 1M + \frac{10}{11} \cdot 1M\right) \quad P \quad \left(\frac{1}{11} \cdot 0M + \frac{10}{11} \cdot 5M\right)$$

iff

$$\frac{11}{100} \cdot \left(\frac{1}{11} \cdot 1M + \frac{10}{11} \cdot 1M\right) + \frac{89}{100} \cdot 0M \quad P \quad \frac{11}{100} \cdot \left(\frac{1}{11} \cdot 0M + \frac{10}{11} \cdot 5M\right) + \frac{89}{100} \cdot 0M$$

iff

$$\left(\frac{1}{100} \cdot 1M + \frac{89}{100} \cdot 0M + \frac{10}{100} \cdot 1M\right) \quad P \quad \left(\frac{1}{100} \cdot 0M + \frac{89}{100} \cdot 0M + \frac{10}{100} \cdot 5M\right)$$

Allais Paradox



	Red (0.01)	White (0.89)	Blue (0.1)
L_1	1M	1M	1M
L_2	0	1M	5M
L_3	1M	0M	1M
L_4	0	0M	5M

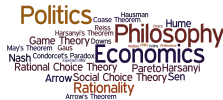
$L_1 P L_2$ if and only if $L_3 P L_4$

Allais Paradox



We should **not** conclude either

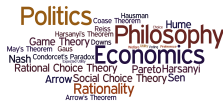
Allais Paradox



We should **not** conclude either

(a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or

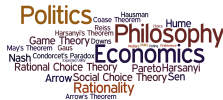
Allais Paradox



We should **not** conclude either

- (a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or
- (b) those who choose L_1 and L_4 are irrational.

Allais Paradox



We should **not** conclude either

- (a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or
- (b) those who choose L_1 and L_4 are irrational.

Rather, people's utility functions (*their rankings over outcomes*) are often far more complicated than the monetary bets would indicate....

L. Buchak (2013). *Risk and Rationality*. Oxford University Press.

Ellsberg Paradox



Lotteries	30	60	
	Blue	Yellow	Green
L_1	1M	0	0
L_2	0	1M	0

Ellsberg Paradox



Lotteries	30	60	
	Blue	Yellow	Green
L_3	1M	0	1M
L_4	0	1M	1M

$$L_1 \text{ } P \text{ } L_2 \text{ if and only if } L_3 \text{ } P \text{ } L_4$$