PHPE 400
Individual and Group Decision Making

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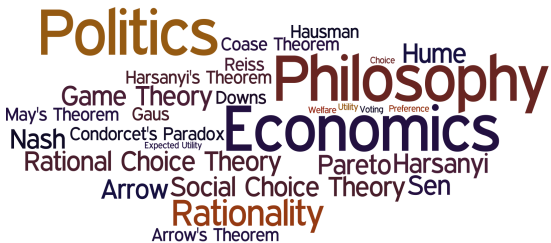

Independence

For all $L_1, L_2, L_3 \in \mathcal{L}$ and $0 < p \leq 1$,

$$L_1 \; P \; L_2 \quad \text{if, and only if,} \quad (p \cdot L_1 + (1-p) \cdot L_3) \; P \; (p \cdot L_2 + (1-p) \cdot L_3).$$

$$L_1 \; I \; L_2 \quad \text{if, and only if,} \quad (p \cdot L_1 + (1-p) \cdot L_3) \; I \; (p \cdot L_2 + (1-p) \cdot L_3).$$

Summary

- For a *cardinal* utility function (measured on an interval scale) that represents a decision maker's preferences over outcomes, the decision maker compares lotteries using **expected utility** based on this utility function.

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Summary

- For a *cardinal* utility function (measured on an interval scale) that represents a decision maker's preferences over outcomes, the decision maker compares lotteries using **expected utility** based on this utility function.
- **Rational preferences** over lotteries are characterized by satisfying Transitivity, Completeness, and the Independence Axiom.
  - If a decision maker violates the Independence Axiom, then no utility function can be used to rank lotteries using expected utility.

Evaluating rational choice axioms

Allais Paradox

	Red (0.01)	White (0.89)	Blue (0.10)
L_1	$1M$	$1M$	$1M$
L_2	0	$1M$	$5M$

Allais Paradox

	Red (0.01)	White (0.89)	Blue (0.10)
L_3	$1M$	0	$1M$
L_4	0	0	$5M$

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Independence and Allais

$$\left(\frac{1}{100}\cdot 1M + \frac{89}{100}\cdot 1M + \frac{10}{100}\cdot 1M\right) \quad P \quad \left(\frac{1}{100}\cdot 0M + \frac{89}{100}\cdot 1M + \frac{10}{100}\cdot 5M\right)$$

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iff

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$L_1 \; P \; L_2$ if and only if $L_3 \; P \; L_4$

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Rather, people's utility functions (*their rankings over outcomes*) are often far more complicated than the monetary bets would indicate....

L. Buchak (2013). *Risk and Rationality*. Oxford University Press.

Ellsberg Paradox

	30	60	
Lotteries	Blue	Yellow	Green
L_1	$1M$	0	0
L_2	0	$1M$	0

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