PHPE 400
Individual and Group Decision Making

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pacuit.org

Independence

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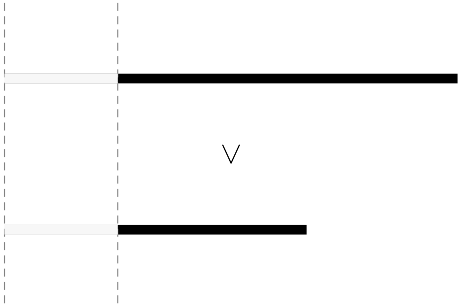
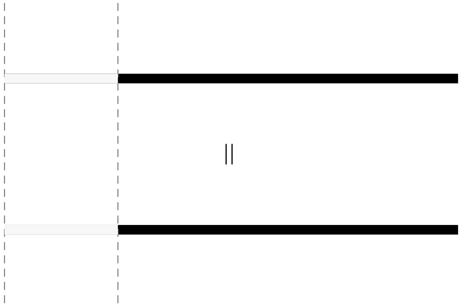
Independence



Independence



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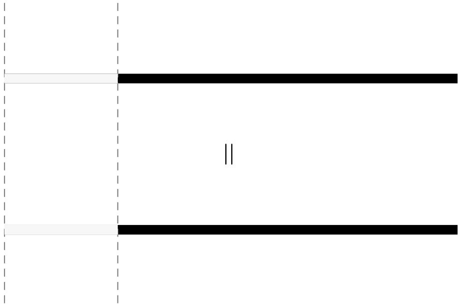

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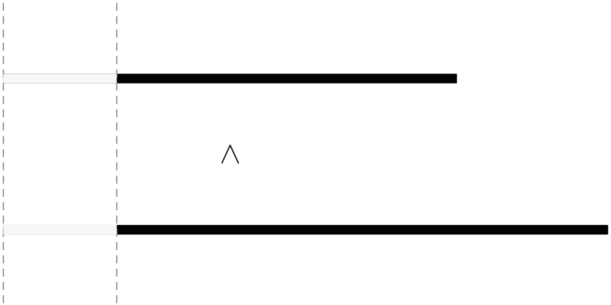

Independence

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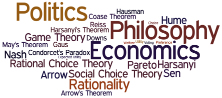
Independence

For all $L_1, L_2, L_3 \in \mathcal{L}$ and $0 < p \leq 1$,

$$L_1 \; P \; L_2 \quad \text{if, and only if,} \quad (p \cdot L_1 + (1-p) \cdot L_3) \; P \; (p \cdot L_2 + (1-p) \cdot L_3).$$

$$L_1 \; I \; L_2 \quad \text{if, and only if,} \quad (p \cdot L_1 + (1-p) \cdot L_3) \; I \; (p \cdot L_2 + (1-p) \cdot L_3).$$

Independence



For all $\color{blue}{\blacksquare}$, $\color{red}{\blacksquare}$, $\color{green}{\blacksquare}$ $\in \mathcal{L}$ and $0 < p \leq 1$,

$$\color{blue}{\blacksquare}\ \color{black}{P}\ \color{red}{\blacksquare} \quad \color{black}{\text{if, and only if,}} \quad (p \cdot \color{blue}{\blacksquare} \color{black}{+ (1-p) \cdot} \color{green}{\blacksquare}\color{black}{)\ P\ (p \cdot} \color{red}{\blacksquare} \color{black}{+ (1-p) \cdot} \color{green}{\blacksquare}\color{black}{).}$$

$$\color{blue}{\blacksquare}\ \color{black}{I}\ \color{red}{\blacksquare} \quad \color{black}{\text{if, and only if,}} \quad (p \cdot \color{blue}{\blacksquare} \color{black}{+ (1-p) \cdot} \color{green}{\blacksquare}\color{black}{)\ I\ (p \cdot} \color{red}{\blacksquare} \color{black}{+ (1-p) \cdot} \color{green}{\blacksquare}\color{black}{).}$$

Example

Consider the set of all lotteries over $X = \{a, b\}$.

Suppose that Ann prefers lotteries that are closer to 50-50. For example,

- $(0.5 \cdot a + 0.5 \cdot b) \;\; P \;\; (0.25 \cdot a + 0.75 \cdot b)$
- $(0.75 \cdot a + 0.25 \cdot b) \;\; I \;\; (0.25 \cdot a + 0.75 \cdot b)$
- $(0.25 \cdot a + 0.75 \cdot b) \;\; P \;\; (1 \cdot a)$

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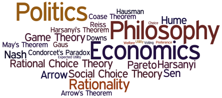

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- $(0.75 \cdot a + 0.25 \cdot b)\ \ I\ \ (0.25 \cdot a + 0.75 \cdot b)$
- $(0.25 \cdot a + 0.75 \cdot b)\ \ P\ \ (1 \cdot a)$

We can view Ann as assigning a value to any lottery as follows:

$$\text{A lottery } r \cdot a + (1-r) \cdot b \text{ is valued at } -|r - \tfrac{1}{2}|.$$

Then, Ann ranks lotteries by assigning a value to the lotteries and ranking them according to the values.

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- Do Ann’s preferences satisfy completeness? Yes.
- Do Ann’s preferences satisfy the Independence Axiom? No.

Ann's preferences violates the Independence Axiom since she has the following preferences:

- $(0.5 \cdot a + 0.5 \cdot b) \;\; P \;\; (1 \cdot a)$
- It is **not** the case that

$$(0.5 \cdot \; (0.5 \cdot a + 0.5 \cdot b) \; + 0.5 \cdot \; b \;) \; P \; (0.5 \cdot \; (1 \cdot a) \; + 0.5 \cdot \; b \;).$$

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$$(0.5 \cdot (0.5 \cdot a + 0.5 \cdot b) + 0.5 \cdot b)\ P\ (0.5 \cdot (1 \cdot a) + 0.5 \cdot b).$$

This is because

$$\begin{aligned}(0.5 \cdot (0.5 \cdot a + 0.5 \cdot b) + 0.5 \cdot b) &= (0.25 \cdot a + 0.75 \cdot b)\\ (0.5 \cdot (1 \cdot a) + 0.5 \cdot b) &= (0.5 \cdot a + 0.5 \cdot b)\end{aligned}$$

And so, $(0.5 \cdot a + 0.5 \cdot b)\ P\ (0.25 \cdot a + 0.75 \cdot b)$ since the value of $(0.5 \cdot a + 0.5 \cdot b)$ is 0 and $(0.25 \cdot a + 0.75 \cdot b)$ is -0.25 and $-0.25 < 0$. Hence, it is not the case that $(0.5 \cdot (0.5 \cdot a + 0.5 \cdot b) + 0.5 \cdot b)\ P\ (0.5 \cdot (1 \cdot a) + 0.5 \cdot b)$.

Violating Independence

A decision maker **does not** satisfy the Independence Axiom when there are lotteries L_1, L_2, L_3 and a number p such that $0 < p \leq 1$ such that at least one of the following is true:

1. $L_1 \; P \; L_2$, but it is not the case that $(p \cdot L_1 + (1-p) \cdot L_3) \; P \; (p \cdot L_2 + (1-p) \cdot L_3)$;
2. $(p \cdot L_1 + (1-p) \cdot L_3) \; P \; (p \cdot L_2 + (1-p) \cdot L_3)$, but it is not the case that $L_1 \; P \; L_2$;
3. $L_1 \; I \; L_2$, but it is not the case that $(p \cdot L_1 + (1-p) \cdot L_3) \; I \; (p \cdot L_2 + (1-p) \cdot L_3)$; or
4. $(p \cdot L_1 + (1-p) \cdot L_3) \; I \; (p \cdot L_2 + (1-p) \cdot L_3)$, but it is not the case that $L_1 \; I \; L_2$;