# PHPE 400
# Individual and Group Decision Making

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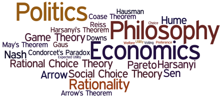
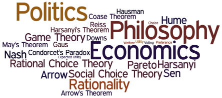
# Representing Preferences




Let $X$ be a set of outcomes. A decision maker's *preference* over $X$ is represented by *relations* on $X$:

- $P \subseteq X \times X$ where $a\ P\ b$ means that the decision maker *strictly prefers* $a$ to $b$.
- $I \subseteq X \times X$ where $a\ I\ b$ means that the decision maker is *indifferent* between $a$ and $b$.
- $N \subseteq X \times X$ where $a\ N\ b$ means that the decision maker *cannot compare* $a$ and $b$.

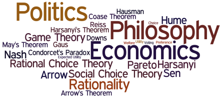
# Preferences - Minimal Constraints




A decision maker's preferences on $X$ is represented by three relations $P \subseteq X \times X$, $I \subseteq X \times X$ and $N \subseteq X \times X$ satisfying the following minimal constraints:

1. For all $x, y \in X$, exactly one of $x\ P\ y$, $y\ P\ x$, $x\ I\ y$ and $x\ N\ y$ is true.
2. $P$ is asymmetric
3. $I$ is reflexive and symmetric.
4. $N$ is symmetric.

# Rational Preferences




An individual's preferences are **rational** when they satisfy two additional constraints:

1. transitivity
2. completeness

# Transitive Relations

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.

**Transitive relation**: for all $x, y, z \in X$, if $x \; R \; y$ and $y \; R \; z$, then $x \; R \; z$

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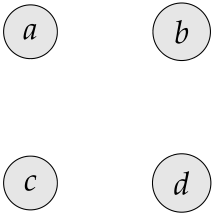

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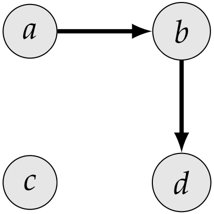

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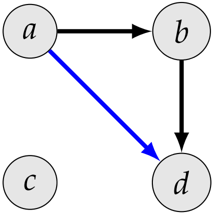

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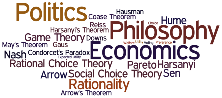


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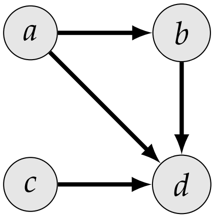

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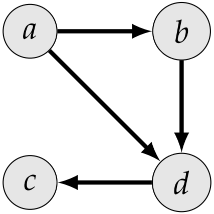

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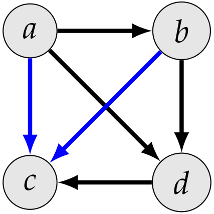

# Transitivity

✓ Strict preference is transitive: for all $x, y, z$ if $x\ P\ y$ and $y\ P\ z$ then $x\ P\ z$

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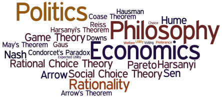
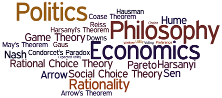



✓ Strict preference is transitive: for all $x, y, z$ if $x \; P \; y$ and $y \; P \; z$ then $x \; P \; z$

? Indifference is transitive: for all $x, y, z$ if $x \; I \; y$ and $y \; I \; z$ then $x \; I \; z$

? Non-comparability is transitive: for all $x, y, z$ if $x \; N \; y$ and $y \; N \; z$ then $x \; N \; z$.

# Transitivity




✗ Indifference: For all $x, y, z \in X$, if $x\ I\ y$ and $y\ I\ z$, then $x\ I\ z$.

- You may be indifferent between a curry with $x$ amount of cayenne pepper, and a curry with $x$ plus one particle of cayenne pepper for any amount $x$. But you are not indifferent between a curry with no cayenne pepper and one with 1 pound of cayenne pepper in it!

# Transitivity

- ✗ Indifference: For all $x, y, z \in X$, if $x \; I \; y$ and $y \; I \; z$, then $x \; I \; z$.
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- ✗ Incomparibility: For all $x, y, z \in X$, if $x \; N \; y$ and $y \; N \; z$, then $x \; N \; z$.
  - You may not be able to compare having a job as a teacher with having a job as lawyer. Furthermore, you cannot compare having a job as a lawyer with having a job as a teacher with an extra \$1,000. However, you do strictly prefer having a job as a teacher with an extra \$1,000 to having a job as a teacher.

# Transitivity

Strict preference is transitive: for all $x, y, z$ if $x \; P \; y$ and $y \; P \; z$ then $x \; P \; z$

# Transitivity

Strict preference is transitive: for all $x, y, z$ if $x \; P \; y$ and $y \; P \; z$ then $x \; P \; z$

There are two ways that a decision maker's strict preference $P$ on $X$ may fail transitivity:

1. There is a **cycle** in the decision maker's preferences: There are $x, y, z \in X$ such that $x \; P \; y$, $y \; P \; z$, and $z \; P \; x$.
2. The decision maker lacks a strict preference: There are $x, y, z \in X$ such that $x \; P \; y$ and $y \; P \; z$, but $x \; N \; z$ (i.e., $x$ and $z$ are incomparable).

# Cyclic Preferences

I do not think we can clearly say what should convince us that [someone] at a given time (without change of mind) preferred *a* to *b*, *b* to *c* and *c* to *a*. The reason for our difficulty is that we cannot make good sense of an attribution of preference except against a background of coherent attitudes...My point is that if we are intelligibly to attribute attitudes and beliefs, or usefully to describe motions as behaviour, then we are committed to finding, in the pattern of behaviour, belief, and desire, a large degree of rationality and consistency. (Davidson 1974: p. 237)

D. Davidson. *'Philosophy as psychology'*. In S. C. Brown (ed.), Philosophy of Psychology, 1974. Reprinted in his Essays on Actions and Events. Oxford: OUP 2001: pp. 229–244.

# Money-Pump Argument

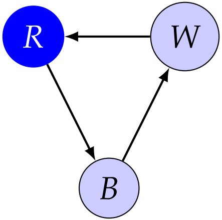
There are three key assumptions about a decision maker's strict preference $P$ and the decision maker's opinion about money:

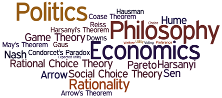
1. If $xPy$, then the decision maker will always take $x$ when $y$ is the only alternative.

2. If $xPy$, then there is some $v > 0$ such that for all $u$, $(x, -\$u)Py$ if and only if $0 \leq u \leq v$.

3. The items and money are *separable* and the decision maker prefers more money to less: For all $x, y \in X$ and $w, z \in \mathbb{R}$, we have that
   - $(x, \$w)P(x, \$z)$ if and only if $w > z$; and,
   - if $xPy$, then $(x, \$w)P(y, \$w)$.

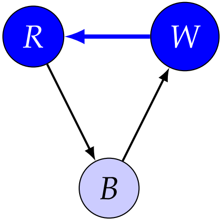
# Money-Pump Argument




$(R)$

# Money-Pump Argument




$$(R) \implies (W, -1)$$

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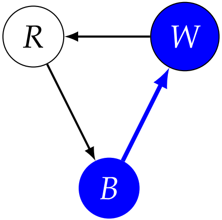
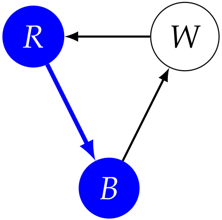



$$(R) \implies (W, -1) \implies (B, -2)$$

# Money-Pump Argument




$$(R) \implies (W, -1) \implies (B, -2) \implies (R, -3) \implies (W, -4) \implies \cdots$$

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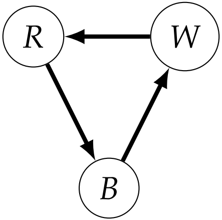


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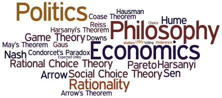


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✗ There is a *cycle* in the decision maker's preferences: There are $x, y, z \in X$ such that $x \; P \; y$, $y \; P \; z$, and $z \; P \; x$.

$\Rightarrow$ Money-pump argument, rankings, ...

1. The decision maker lacks a strict preference: There are $x, y, z \in X$ such that $x \; P \; y$ and $y \; P \; z$, but $x \; N \; z$ (i.e., $x$ and $z$ are incomparable).

# Completeness

Completeness: For all $x, y \in X$, exactly one the following is true:

- $x \; P \; y$ ($x$ is strictly preferred to $y$),
- $y \; P \; x$ ($y$ is strictly preferred to $x$), or
- $x \; I \; y$ ($x$ and $y$ are indifferent).

I.e., for all $x, y \in X$, not-$x \; N \; y$.