PHPE 400
Individual and Group Decision Making

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The concept of "preference" is central to economic theory. Economists typically take preferences to be predetermined or "given" facts about individuals and, for their purposes, not in need of explanation or subject to substantive appraisal. (p. 56, Hausman, McPherson and Satz)

Preferences

Preferring or choosing *x* is different that "liking" *x* or "having a taste for *x*": one can prefer *x* to *y* but *dislike* both options

Preferences are always understood as *comparative*: "preference" is more like "bigger" than "big"

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Mathematical background: Relations

Suppose that X is a set.

An **ordered pair** of elements from X is (a, b) where $a \in X$ is the first component and $b \in X$ is the second component.

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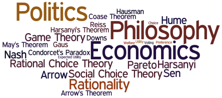

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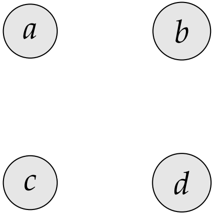
A **relation** on X is a set of **ordered pairs** from X.

That is, if R is a relation on X, then $R \subseteq X \times X$.

Mathematical background: Relations

Example: $X = \{a, b, c, d\}$, $R = \{(a, a), (b, a), (c, d), (a, c), (d, d)\}$



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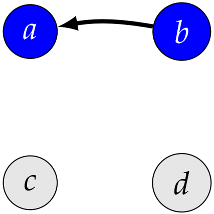

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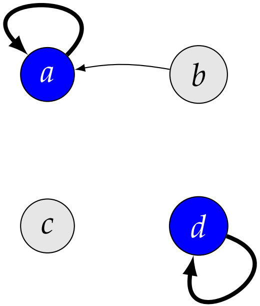

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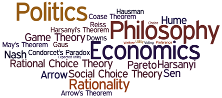

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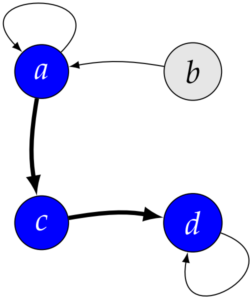

$a \; R \; a$

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$c \; R \; d$

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Strict Preference

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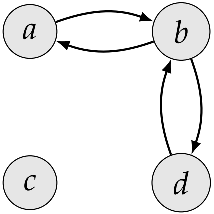
Can *any* relation on X represent a strict preference for a decision maker?

Symmetric/Asymmetric Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Symmetric relation: for all $x, y \in X$, if $x \mathrel{R} y$, then $y \mathrel{R} x$

Asymmetric relation: for all $x, y \in X$, if $x \mathrel{R} y$, then not-$y \mathrel{R} x$



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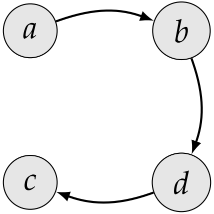

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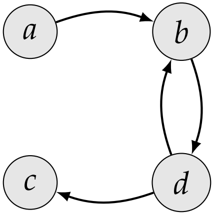

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Strict Preference

A decision maker's strict preference over a set X is represented as a relation $P \subseteq X \times X$.

The underlying idea is that if P represents the decision maker's strict preference and $x\ P\ y$ (i.e., the decision maker strictly prefers x to y), then the decision maker would pay some non-zero amount money to trade y for x.

Assumption: P is asymmetric (for all $x, y \in X$, if $x\ P\ y$, then it is not the case that $y\ P\ x$, written not-$y\ P\ x$).

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There are two reasons why this might hold:

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   In this case, we write $x\ I\ y$.
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What properties should I and N satisfy?

Reflexive Relations

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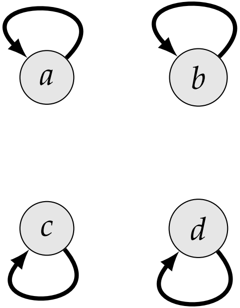
Reflexive relation: for all $x \in X$, $x \, R \, x$



Representing Preferences

Let X be a set of outcomes. A decision maker's *preference* over X is represented by *relations* on X:

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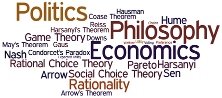
Preferences - Minimal Constraints



A decision maker's preferences on X is represented by three relations $P \subseteq X \times X$, $I \subseteq X \times X$ and $N \subseteq X \times X$ satisfying the following minimal constraints:

1. For all $x, y \in X$, exactly one of $x\ P\ y$, $y\ P\ x$, $x\ I\ y$ and $x\ N\ y$ is true.
2. P is asymmetric
3. I is reflexive and symmetric.
4. N is symmetric.