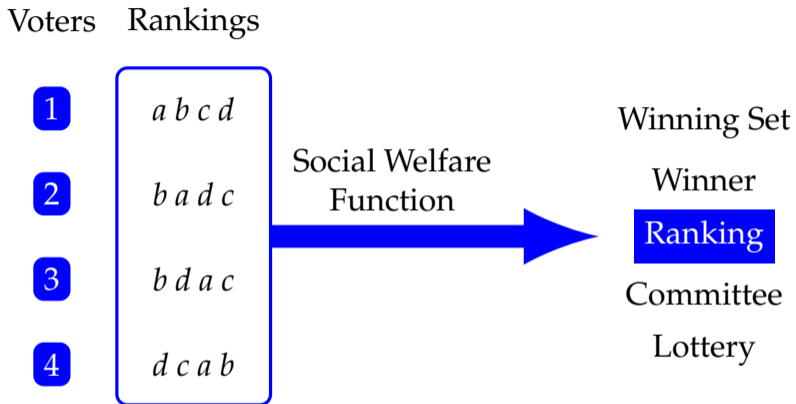


PHPE 400

Individual and Group Decision Making

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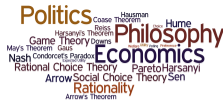
Arrow's Theorem



Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies Universal Domain, Rationality, Independence of Irrelevant Alternatives (IIA) and Pareto is a dictatorship.

- ▶ Alternative statement of the theorem: Suppose that there are at least three candidates and finitely many voters. There is no social welfare function that satisfies Universal Domain, Rationality, Independence of Irrelevant Alternatives (IIA), Pareto, and Non-Dictatorship.

Evaluative Voting



In Arrow's theorem, it is assumed that the input is the voters' *rankings* of the candidates.

One response to Arrow's theorem is to ask for *more information* from the voters about their opinions of the candidates.

Approval Voting bridges America's divide.

A **simple solution** to repair our democracy that is supported by over 70% of the public!

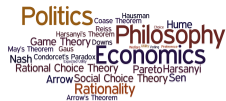


<https://electionscience.org>

Approval Voting: Each voter selects a subset of candidates. The candidate with the most “approvals” wins the election.

S. Brams and P. Fishburn. *Approval Voting*. Birkhauser, 1983.

J.-F. Laslier and M. R. Sanver (eds.). *Handbook of Approval Voting*. Studies in Social Choice and Welfare, 2010.



Under Approval Voting (AV), voters are asked to select the candidates that the voter *approves*.

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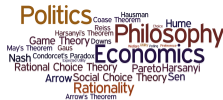
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The two pieces of information are related, but not derivable from each other

Approving of a candidate is not (necessarily) the same as simply ranking the candidate first.

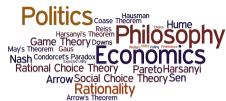
Generalizing Approval Voting



In many group decision situations, people use measures or grades from a **common language of evaluation** to evaluate candidates or alternatives:

- ▶ in figure skating, diving and gymnastics competitions;
- ▶ in piano, flute and orchestra competitions;
- ▶ in classifying wines at wine competitions;
- ▶ in ranking university students;
- ▶ in classifying hotels and restaurants, e.g., the Michelin *

Score Voting/Range Voting



Governor Candidates		Score <i>each</i> candidate by filling a number (0 is worst; 9 is best)
1: Candidate A	→	0 1 2 3 4 5 6 7 8 9
2: Candidate B	→	0 1 2 3 4 5 6 7 8 9
3: Candidate C	→	0 1 2 3 4 5 6 7 8 9

Score Voting/Range Voting



Fixe a common grading language consisting of, for example, the integers $\{0, 1, 2, \dots, 10\}$

The candidate with the largest *average* grade is declared the winner.

MAJORITY JUDGMENT

Measuring, Ranking, and Electing



MICHEL BALINSKI AND RIDA LARAKI

Majority Judgment



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Majority Judgment

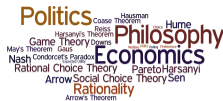


Majority Judgment is like Score Voting but picks the candidate with the greater *median* score instead of the greatest average score.

1. Each voter assigns each candidate a score from the set $\{0, \dots, n\}$;
2. The candidate with the greatest median score wins according to Majority Judgement.

Note: if there is an even number of voters, Majority Judgement uses the “lower median.” E.g., if the scores for A are 7, 7, 8, 8, 11, 11, 11, 13, the lower median is 8.

Evaluative Voting

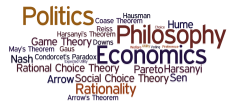


Approval Voting: voters can assign a single grade “approve” to the candidates. The candidates with the most approvals are the winner.

Score Voting: voters can assign any grade from a fixed set of grades to the candidates. The candidate with the greatest sum of the grades is the winner.

Majority Judgement: voters can assign any grade from a fixed set of grades to the candidates. The candidate with the greatest median grade is the winner.

Grading vs. Ranking



S. Brams and R. Potthoff. *The paradox of grading systems*. *Public Choice*, 165, pp. 193 - 210, 2015.

Grading vs. Ranking



Suppose that the possible grades are $\{0, 1, \dots, 20\}$

# of Voters	<i>A</i>	<i>B</i>
1	20	11
1	9	0
1	9	10
Median:	9	10

Majority Judgement Winner: *B*

Grading vs. Ranking



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# of Voters	<i>A</i>	<i>B</i>
1	20	11
1	9	0
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Median:	9	10

Majority Judgement Winner: *B*

2 out of 3 voters prefer *A* to *B*

Grading vs. Ranking



Suppose that the possible grades are $\{0, 1, \dots, 20\}$

# of Voters	<i>A</i>	<i>B</i>
50	20	11
50	9	0
1	9	10
Median:	9	10

Majority Judgement Winner: *B*

Grading vs. Ranking



Suppose that the possible grades are $\{0, 1, \dots, 20\}$

# of Voters	<i>A</i>	<i>B</i>
50	20	11
50	9	0
1	9	10
Median:	9	10

Majority Judgement Winner: *B*

100 out of 101 voters prefer *A* to *B*

Grades: $\{0, 1, 2, 3, 4, 5\}$

Candidates: $\{A, B, C\}$

5 Voters

# of Voters	<i>A</i>	<i>B</i>	<i>C</i>
1	5	0	0
4	0	1	1
Mean:	1	4/5	4/5

Grades: $\{0, 1, 2, 3, 4, 5\}$

Candidates: $\{A, B, C\}$

5 Voters

# of Voters	A	B	C
1	5	0	0
4	0	1	1
Mean:	1	4/5	4/5

Average Grade Winner: A

Superior Grade Winner: B, C

The Preference Intensity Problem



$$\begin{array}{r} 51 \quad 49 \\ \hline a \quad b \\ b \quad a \end{array}$$

51% of the voters have a *slight* preference for a over b and 49% of the voters have a *strong* preference for b over a .

Should candidate a win the election?

The Preference Intensity Problem



80	20
<hr/>	
a	b
b	a

80% of the voters *strictly prefer* a over b and 20% of the voters have an “*extremely strong*” preference for b over a .

Should candidate a win the election?

The Preference Intensity Problem



$$\begin{array}{cc} 75 & 25 \\ \hline a & b \\ b & a \end{array}$$

75% of the voters *strictly prefer a* over *b* and 25% of the voters *strictly prefer b* over *a*. If *a* wins, then this will cause harm to the 25% of voters that prefer *b* to *a*; and if *b* wins, this will cause some annoyance to the 75% of the voters that prefer *a* to *b*.

How do we weigh the preference of the majority while avoiding harm to the minority?

The Preference Intensity Problem



$$\begin{array}{cc} 75 & 25 \\ \hline a & b \\ b & a \end{array}$$

75% of the voters *strictly prefer* a over b and 25% of the voters *strictly prefer* b over a . If a wins, then this will cause harm to the 25% of voters that prefer b to a ; and if b wins, this will cause some annoyance to the 75% of the voters that prefer a to b .

How do we weigh the preference of the majority while avoiding harm to the minority?

- ▶ Not all questions should be decided by a vote.
- ▶ Education, deliberation, etc. to change the rankings of the enough of the 75% of the voters to ensure that b is the majority opinion.

Systematic Minority



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- ▶ When group barriers are permeable, the minority can occasionally belong to the winning side. However, when preferences are fully polarized and the power of a cohesive majority bloc is secure, the minority is consistently outvoted.
- ▶ Some solutions:
 - ▶ Ensure that the political districts are *fair* (see <https://mggg.org/>).
 - ▶ In some cases, power-sharing is imposed through constitutional provisions that allocate positions to specific ethnic or religious groups. However, such measures are often heavy-handed and difficult to enforce.