# Arrow's theorem

Politics Coase Theorem Hume Philosophy
Harsanyi's Theorem Rolls
Game Theory Downs
May's Theorem Gaus
Nash Condorcet's Paradox Economics
Rational Choice Theory Pareto Harsanyi
Arrow Social Choice Theory Sen
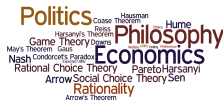Rationality
Arrow's Theorem

**Amartya Sen's Version of Arrow's Theorem**. Assume $X$ is a set of candidates with at least 3 elements and that $V$ is finite. There is no SWFL $f$ satisfying **Universal Domain**, **Pareto**, **Cardinal Invariance**, **Independence of Irrelevant Utilities**, **Rationality**, and **Non-Dictatorship**.

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One response to Arrow's Theorem is to drop Cardinal Invariance in favor of, for example, the sum utilitarian social welfare functional that requires **interpersonal comparisons of utility**.

# Against Interpersonal Comparisons of Utility

**Arrow**: "...It requires a definite value judgment not derivable from individual sensations to make the utilities of different individuals dimensionally compatible and still a further value judgment to aggregate them according to any particular mathematical formula.

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**Arrow**: "...It requires a definite value judgment not derivable from individual sensations to make the utilities of different individuals dimensionally compatible and still a further value judgment to aggregate them according to any particular mathematical formula.

If we look away from the mathematical aspects of the matter, it seems to make no sense to add the utility of one individual, a psychic magnitude in his mind, with the utility of another individual. Even Bentham had his doubts on this point." *(Social Choice and Individual Values*, p. 11).

# Example

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Mary      seashore $P$ museums $P$ camping
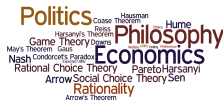
Sam       camping $P$ museums $P$ seashore

- ▶ The seashore is the only alternative that Mary finds bearable, although she feels more negative about going to the mountains than to the museums.
- ▶ Each choice is fine with Sam, although he would much prefer going to the mountains.

|           | Mary | Sam | Total |
|-----------|------|-----|-------|
| Seashore  | 20   |     |       |
| Museums   | 10   |     |       |
| Mountains | 9    |     |       |

|           | Mary | Sam | Total |
|-----------|------|-----|-------|
| Seashore  | 20   | 86  |       |
| Museums   | 10   | 93  |       |
| Mountains | 9    | 100 |       |

|            | Mary | Sam | Total |
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| Seashore   | 20   | 86  | 106   |
| Museums    | 10   | 93  | 103   |
| Mountains  | 9    | 100 | 109   |

# Example

|  | Mary | Sam | Total |
|---|---|---|---|
| Seashore | 20 | 86 | 106 |
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For Mary, the difference between the seashore and the mountains crosses the threshold between the bearable and the intolerable. She feels that her "right to an emotionally recuperative vacation" will be violated by following a utilitarian scheme.

# Example

|           | Mary | Sam | Total |
|-----------|------|-----|-------|
| Seashore  | 200  | 86  | 286   |
| Museums   | 100  | 93  | 190   |
| Mountains | 90   | 100 | 190   |

Mary: My preferences are so intense in comparison with yours that my scale should range between 0 and 1,000, if yours range between 0 and 100.
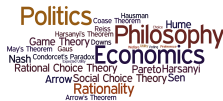
# Example

| | Mary | Sam | Total |
|---|---|---|---|
| Seashore | 20 | 86 | 106 |
| Museums | 10 | 93 | 103 |
| Mountains | 9 | 100 | 109 |

Sam: You think that my preferences are rather weak, but the fact is I feel things quite deeply. I have been brought up in a culture very different from yours and have been trained to avoid emotional outbursts...But I have strong feelings all the same.

# Example

|           | Mary | Sam | Total |
|-----------|------|-----|-------|
| Seashore  | 20   | 86  | 106   |
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Sam: I do not think that extra weight *should* be given in a utilitarian calculation to those who are capable of more intense preferences.

- ▶ Is Mary's preference for the seashore *really* stronger than Sam's for the mountains? Or, is Mary just a more vocal person?

- ▶ If some people's preferences are in fact stronger than others', how could we *know* this?

- ▶ Does it make any more sense to compare Sam's preferences with Mary's than it does to compare a dog's preference for steak bones with a horse's preference for oats?

- ▶ Even if we answer all these questions affirmatively, is it morally proper to respond to such information in making social choices?

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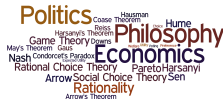
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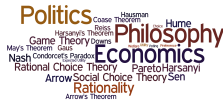
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We already discussed the epistemological objection.

# A standard moral objection

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Let $x$ be the alternative that the robot leaves Alice and Bob alone. In this case, $\mathbf{U}_a(x) = 5$ and $\mathbf{U}_b(x) = 6$.

Let $y$ be the alternative that the robot pushes Alice to the ground for Bob's enjoyment. In this case, $\mathbf{U}_a(y) = 2$ and $\mathbf{U}_b(y) = 10$.

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The fundamental moral problem is that Utilitarianism allows the robot to **use one person as a resource** to increase total utility.
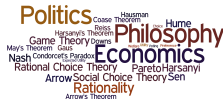
# Utility monsters

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There is another problem in the Alice-Bob example, in addition to the Utilitarian robot using Alice as a resource to benefit Bob.
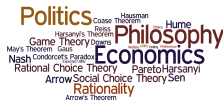
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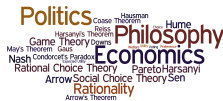
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# Utility monsters

There is another problem in the Alice-Bob example, in addition to the Utilitarian robot using Alice as a resource to benefit Bob.

It is that Bob enjoys a utility gain from watching another person suffer. **According to Utilitarianism, such a utility gain cannot be ignored.**

# Utility monsters

**Nozick**: "Utilitarian theory is embarrassed by the possibility of **utility monsters** who get enormously greater gains in utility from any sacrifice of others than these others lose. For, unacceptably, the theory seems to require that we all be satisfied in the monster's maw, in order to increase total utility" (*Anarchy, State, and Utopia*, p. 41).

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Of course, a Utilitarian could claim that in the actual world, there are no utility monsters—raising again the epistemological problem of how they know this—while of course admitting, true to their doctrine, that if there were utility monsters, we *should* sacrifice for them.

# Anti-Aggregationism
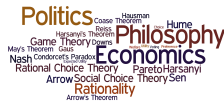


Suppose that you have a choice between:

1. Cure one young person of a terminal illness.
2. Cure $n$ young people of a mild illness that will cause them to have a mild headache for one day.

How large must $n$ be to justify choosing the second option?

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**Anti-Aggregationism**: there is no $n$ that would ever justify the second option.

Thank you!