PHPE 400
Individual and Group Decision Making

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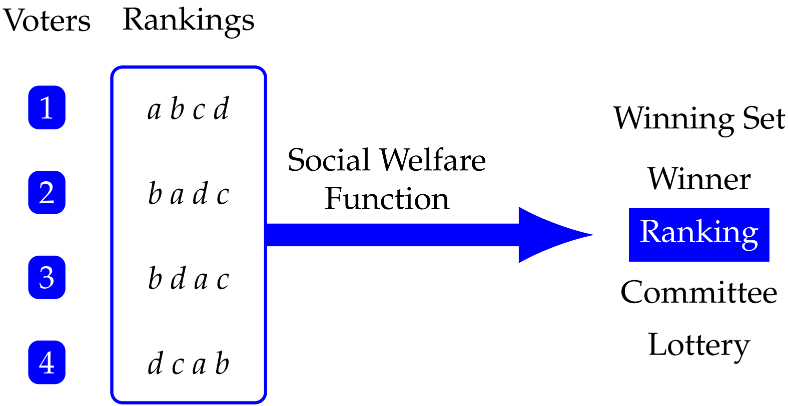
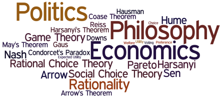

Voters
Rankings
1
2
3
4
$a\ b\ c\ d$
$b\ a\ d\ c$
$b\ d\ a\ c$
$d\ c\ a\ b$
Social Welfare Function
Winning Set
Winner
Ranking
Committee
Lottery

Arrow’s Theorem



Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies Universal Domain, Rationality, Independence of Irrelevant Alternatives (IIA) and Pareto is a dictatorship.

- Alternative statement of the theorem: Suppose that there are at least three candidates and finitely many voters. There is no social welfare function that satisfies Universal Domain, Rationality, Independence of Irrelevant Alternatives (IIA), Pareto, and Non-Dictatorship.

Evaluative Voting

In Arrow's theorem, it is assumed that the input is the voters' *rankings* of the candidates.

One response to Arrow's theorem is to ask for more information from the voters about their opinions of the candidates.

https://electionscience.org

Approval Voting: Each voter selects a subset of candidates. The candidate with the most "approvals" wins the election.

S. Brams and P. Fishburn. *Approval Voting*. Birkhauser, 1983.

J.-F. Laslier and M. R. Sanver (eds.). *Handbook of Approval Voting*. Studies in Social Choice and Welfare, 2010.

Under Approval Voting (AV), voters are asked to select the candidates that the voter *approves*.

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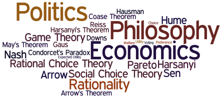

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Approving of a candidate is not (necessarily) the same as simply ranking the candidate first.

Why Approval Voting?

https://electionscience.org

S. Brams and P. Fishburn. *Going from Theory to Practice: The Mixed Success of Approval Voting*. Handbook of Approval Voting, pp. 19-37, 2010.

Approval Voting is more flexible

# voters	2	2	1
	a	b	c
	d	d	a
	b	a	b
	c	c	d

The Condorcet winner is a.

Approval Voting is more flexible

There is no fixed rule that always elects a unique Condorcet winner.

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Vote-for-1 elects $\{a, b\}$

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Vote-for-1 elects $\{a, b\}$, vote-for-2 elects $\{d\}$

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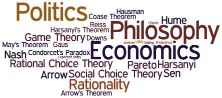

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# voters	2	2	1
	a	b	c
	d	d	a
	b	a	b
	c	c	d

The Condorcet winner is a.
Vote-for-1 elects $\{a, b\}$, vote-for-2 elects $\{d\}$, vote-for-3 elects $\{a, b\}$.

Approval Voting is more flexible

AV may elect the Condorcet winner

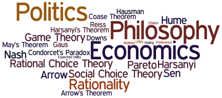
# voters	2	2	1
	a	b	c
	d	d	a
	b	a	b
	c	c	d

The Condorcet winner is a.
$(\{a\}, \{b\}, \{c, a\})$ elects a under AV.

Possible Failure of Unanimity

# voters	1	1	1
	a	c	d
	b	a	a
	c	b	b
	d	d	c

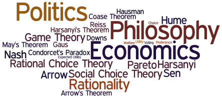
Possible Failure of Unanimity



# voters	1	1	1
	a	*c*	*d*
	b	*a*	*a*
	c	*b*	*b*
	d	*d*	*c*

Approval Winners: a, b

Generalizing Approval Voting



In many group decision situations, people use measures or grades from a **common language of evaluation** to evaluate candidates or alternatives:

- in figure skating, diving and gymnastics competitions;
- in piano, flute and orchestra competitions;
- in classifying wines at wine competitions;
- in ranking university students;
- in classifying hotels and restaurants, e.g., the Michelin $*$

Score Voting/Range Voting

Governor Candidates		Score *each* candidate by filling a number (0 is worst; 9 is best)									
1: Candidate A	→	**0**	1	2	3	4	5	6	7	8	9
2: Candidate B	→	0	1	2	3	4	5	6	7	8	**9**
3: Candidate C	→	0	1	2	3	4	5	6	**7**	8	9

Score Voting/Range Voting

Fixe a common grading language consisting of, for example, the integers $\{0, 1, 2, \ldots, 10\}$

The candidate with the largest *average* grade is declared the winner.

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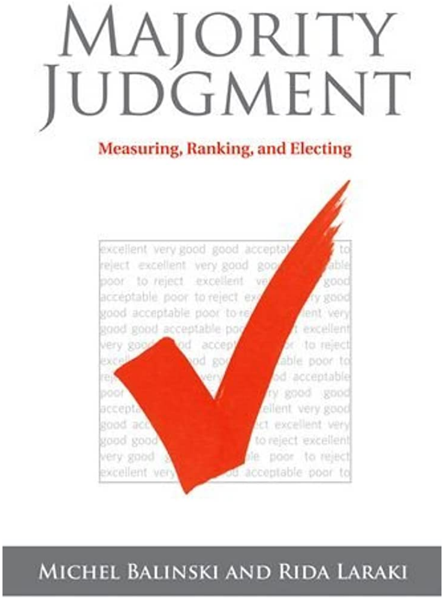
The candidate with the largest *average* grade is declared the winner.

Suppose a's grades are $\{7, 7, 8, 8, 9, 9, 9, 10\}$. The average grade is 8.375

Suppose b's grades are $\{9, 9, 9, 9, 9, 10, 10, 10\}$. The average grade is 9.375

So, Score Vote (Range Vote) ranks b above candidate a.


MAJORITY JUDGMENT
Measuring, Ranking, and Electing
MICHEL BALINSKI AND RIDA LARAKI

Majority Judgment

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1. Each voter assigns each candidate a score from the set $\{0, \ldots, n\}$;
2. The candidate with the greatest median score wins according to Majority Judgement.

Note: if there is an even number of voters, Majority Judgement uses the "lower median." E.g., if the scores for A are $7, 7, 8, 8, 11, 11, 11, 13$, the lower median is 8.

Evaluative Voting

Approval Voting: voters can assign a single grade "approve" to the candidates. The candidates with the most approvals are the winner.

Score Voting: voters can assign any grade from a fixed set of grades to the candidates. The candidate with the greatest sum of the scores is the winner.

Majority Judgement: voters can assign any grade from a fixed set of grades to the candidates. The candidate with the greatest median score is the winner.

Score Voting vs. Majority Judgement

Consider the following example from the SEP entry on “Voting Methods”:

# of Voters	A	B	C
1	4	3	1
1	4	3	2
1	2	0	3
1	2	3	4
1	1	0	2
Mean:	2.6	1.8	2.4
Median:	2	3	2

Score Voting vs. Majority Judgement

Consider the following example from the SEP entry on "Voting Methods":

# of Voters	A	B	C
1	4	3	1
1	4	3	2
1	2	0	3
1	2	3	4
1	1	0	2
Mean:	2.6	1.8	2.4
Median:	2	3	2

Thus, *A* wins according to Score Voting, while *B* wins according to Majority Judgement.

Score Voting vs. Majority Judgement

Here is another example from the *Majority Voting* book (p. 282) showing how Majority Judgement differs from Score Voting:

# of Voters	A	B
k	20	20
1	19	20
k	19	0
Mean:	slightly under 19.5	slightly over 10
Median:	19	20

Grading vs. Ranking

S. Brams and R. Potthoff. *The paradox of grading systems*. Public Choice, 165, pp. 193 - 210, 2015.

Grading vs. Ranking

Suppose that the possible grades are $\{0, 1, \ldots, 20\}$

# of Voters	*A*	*B*
1	20	11
1	9	0
1	9	10
Median:	9	10

Majority Judgement Winner: *B*

Grading vs. Ranking

Suppose that the possible grades are $\{0, 1, \ldots, 20\}$

# of Voters	A	B
1	20	11
1	9	0
1	9	10
Median:	9	10

Majority Judgement Winner: B

2 out of 3 voters prefer A to B

Grading vs. Ranking

Suppose that the possible grades are $\{0, 1, \ldots, 20\}$

# of Voters	A	B
50	20	11
50	9	0
1	9	10
Median:	9	10

Majority Judgement Winner: B

Grading vs. Ranking

Suppose that the possible grades are $\{0, 1, \ldots, 20\}$

# of Voters	A	B
50	20	11
50	9	0
1	9	10
Median:	9	10

Majority Judgement Winner: B

100 out of 101 voters prefer A to B

Grades: $\{0, 1, 2, 3, 4, 5\}$
Candidates: $\{A, B, C\}$
5 Voters

# of Voters	A	B	C
1	5	0	0
4	0	1	1
Mean:	1	4/5	4/5

Grades: $\{0, 1, 2, 3, 4, 5\}$
Candidates: $\{A, B, C\}$
5 Voters

# of Voters	A	B	C
1	5	0	0
4	0	1	1
Mean:	1	4/5	4/5

Average Grade Winner: A

Superior Grade Winner: B, C

To conclude, we have identified a paradox of grading systems, which is not just a mirror of the well-known differences that crop up in aggregating votes under ranking systems. Unlike these systems, for which there is no accepted way of reconciling which candidate to choose when, for example, the Hare, Borda and Condorcet winners differ, AV provides a solution when the AG and SG winners differ.

Theorem (Brams and Potthoff). When there are two grades, the AG and SG winners are identical.

The Preference Intensity Problem

51	49
a	b
b	a

51% of the voters have a *slight* preference for a over b and 49% of the voters have a *strong* preference for b over a.

Should candidate a win the election?

The Preference Intensity Problem

80	20
a	b
b	a

80% of the voters *strictly prefer* a over b and 20% of the voters have an "*extremely strong*" preference for b over a.

Should candidate a win the election?

The Preference Intensity Problem

75	25
a	b
b	a

75% of the voters *strictly prefer* a over b and 25% of the voters *strictly prefer* b over a. If a wins, then this will cause harm to the 25% of voters that prefer b to a; and if b wins, this will cause some annoyance to the 75% of the voters that prefer a to b.

How do we weigh the preference of the majority while avoiding harm to the minority?

The Preference Intensity Problem

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a	b
b	a

75% of the voters *strictly prefer* a over b and 25% of the voters *strictly prefer* b over a. If a wins, then this will cause harm to the 25% of voters that prefer b to a; and if b wins, this will cause some annoyance to the 75% of the voters that prefer a to b.

How do we weigh the preference of the majority while avoiding harm to the minority?

- Not all questions should be decided by a vote.
- Education, deliberation, etc. to change the rankings of the enough of the 75% of the voters to ensure that b is the majority opinion.

Systematic Minority

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- When group barriers are permeable, the minority can occasionally belong to the winning side.
- When preferences are fully polarized and the power of a cohesive majority bloc is secure, the minority remains disenfranchised.
- Some solutions:
  - Ensure that the political districts are *fair*: `https://mggg.org/`
  - In some instances power-sharing is imposed directly, and the constitution grants executive positions to specific groups, typically on the basis of their ethnic or religious identity. The problem is that constitutional provisions of this type are difficult to enforce and heavy-handed.