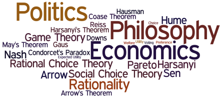
PHPE 400
Individual and Group Decision Making

Eric Pacuit
University of Maryland
pacuit.org

Judgement Aggregation Paradoxes

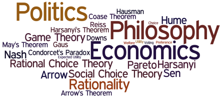


Kornhauser and Sager. *Unpacking the court*. Yale Law Journal, 1986.

P. Mongin. *The doctrinal paradox, the discursive dilemma, and logical aggregation theory*. Theory and Decision, 73(3), pp 315 - 355, 2012.

C. List and P. Pettit. *Aggregating sets of judgments: An impossibility result*. Economics and Philosophy 18, pp. 89 - 110, 2002.

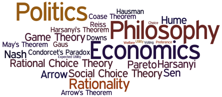
Judgement Aggregation Paradox



Should we hire the candidate?

- Is the candidate good at research (r)?
- Is the candidate good at teaching (t)?
- We should hire the candidate if and only if the candidate is good at research and teaching. ($r \wedge t$)

Judgement Aggregation Paradox



Is the candidate good at research (r)? Is the candidate good at teaching (t)?

Should we hire the candidate (h)?

	r	t		h
Voter 1				
Voter 2				
Voter 3				
Group				

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

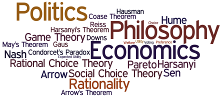
	r	t		h
Voter 1	Yes	Yes		
Voter 2	Yes	No		
Voter 3	No	Yes		
Group	Yes	Yes		

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes		
Voter 2	Yes	No		
Voter 3	No	Yes		
Group	Yes	Yes	Yes	Yes

Judgement Aggregation Paradox



Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes	Yes	Yes
Voter 2	Yes	No	Yes	No
Voter 3	No	Yes	Yes	No
Group				No

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?

Should we hire the candidate (h)?

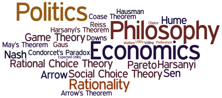
	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes	Yes	Yes
Voter 2	Yes	No	Yes	No
Voter 3	No	Yes	Yes	No
Group	Yes	Yes	Yes	Y/N

What happens when there are more than 2 candidates?

- ✓ Group decision problems often exhibit a *combinatorial structure*. For example, voting on a number of yes/no issues in a referendum, or voting on different interconnected issues.
- ► As we have seen, there are many different reasonable voting methods that generalize Majority Rule for more than 2 candidates.

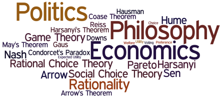
  Is there a voting method that satisfies *all* principles of group decision making?

Principles of group decision making



- **Anonymity**: If voters swap their ballots, then the outcome is unaffected.
- **Neutrality**: If candidates are exchanged in every ranking, then the outcome changes accordingly.
- **Resoluteness**: Always elect a single winner.

Condorcet Triples and Resoluteness



n	n	n
a	b	c
b	c	a
c	a	b

n	n	n
a	c	b
c	b	a
b	a	c

Fact. In both profiles, any voting method satisfying anonymity and neutrality must select all candidates as winners

1	1	1
a	b	c
b	c	a
c	a	b

Consider $\mathbf{P} = (a\ b\ c, b\ c\ a, c\ a\ b)$ and suppose that $F(a\ b\ c, b\ c\ a, c\ a\ b) = \{a\}$

Suppose that $F(a\ b\ c,\ b\ c\ a,\ c\ a\ b) = \{a\}$

Suppose that $F(a\ b\ c, b\ c\ a, c\ a\ b) = \{a\}$

1. Swap a and b in everyone's rankings in the given profile. Then, by Neutrality:
$$F(b\ a\ c, a\ c\ b, c\ b\ a) = \{b\}$$

Suppose that $F(a\ b\ c, b\ c\ a, c\ a\ b) = \{a\}$

1. Swap a and b in everyone's rankings in the given profile. Then, by Neutrality:
$$F(b\ a\ c, a\ c\ b, c\ b\ a) = \{b\}$$

2. Swap b and c in everyone's rankings in the profile from step 1. Then, by Neutrality:
$$F(c\ a\ b, a\ b\ c, b\ c\ a) = \{c\}$$

Suppose that $F(a\ b\ c, b\ c\ a, c\ a\ b) = \{a\}$

1. Swap a and b in everyone's rankings in the given profile. Then, by Neutrality:
$$F(b\ a\ c, a\ c\ b, c\ b\ a) = \{b\}$$
2. Swap b and c in everyone's rankings in the profile from step 1. Then, by Neutrality:
$$F(c\ a\ b, a\ b\ c, b\ c\ a) = \{c\}$$
3. By Anonymity, the original profile and the profile in step 3 must have the same winners:
$$F(a\ b\ c, b\ c\ a, c\ a\ b) = F(c\ a\ b, a\ b\ c, b\ c\ a)$$

Suppose that $F(a\ b\ c, b\ c\ a, c\ a\ b) = \{a\}$

1. Swap a and b in everyone's rankings in the given profile. Then, by Neutrality:
$$F(b\ a\ c, a\ c\ b, c\ b\ a) = \{b\}$$

2. Swap b and c in everyone's rankings in the profile from step 1. Then, by Neutrality:
$$F(c\ a\ b, a\ b\ c, b\ c\ a) = \{c\}$$

3. By Anonymity, the original profile and the profile in step 3 must have the same winners:
$$F(a\ b\ c, b\ c\ a, c\ a\ b) = F(c\ a\ b, a\ b\ c, b\ c\ a)$$

4. 1 and 2 contradict 3 since
$F(a\ b\ c, b\ c\ a, c\ a\ b) = \{a\} \neq \{c\} = F(c\ a\ b, a\ b\ c, b\ c\ a)$.

So, tie-breaking cannot be built-in to a voting method: there is no voting method that satisfies Anonymity, Neutrality and always elects a single winner.

Recall Weak Positive Responsiveness

- F satisfies **weak positive responsiveness** if for any profiles $\mathbf{P}$ and $\mathbf{P}'$, if

  1. $a \in F(\mathbf{P})$ (a is a winner in $\mathbf{P}$ according to F) and
  2. $\mathbf{P}'$ is obtained from $\mathbf{P}$ by one voter who ranked a uniquely last in $\mathbf{P}$ switching to ranking a uniquely first in $\mathbf{P}'$,

  then $F(\mathbf{P}') = \{a\}$ (a is the **unique** winner in $\mathbf{P}'$ according to F).

Monotonicity

A candidate receiving more “support” shouldn’t maker her worse off.

Monotonicity

A candidate receiving more "support" shouldn't maker her worse off.

More-is-Less Paradox: If a candidate c is elected under a given a profile of rankings of the competing candidates, it is possible that, *ceteris paribus*, c may not be elected if some voter(s) raise c in their rankings.

P. Fishburn and S. Brams. *Paradoxes of Preferential Voting*. Mathematics Magazine (1983).

More-is-Less Paradox: Ranked Choice

6	5	4	2
a	c	b	b
b	a	c	a
c	b	a	c

6	5	4	2
a	c	b	a
b	a	c	b
c	b	a	c

More-is-Less Paradox: Ranked Choice

6	5	4	2
a	*c*	*b*	*b*
b	*a*	*c*	*a*
c	*b*	*a*	*c*

6	5	4	2
a	*c*	*b*	*a*
b	*a*	*c*	*b*
c	*b*	*a*	*c*

More-is-Less Paradox: Ranked Choice

6	5	4	2
a	c	b	b
b	a	c	a
c	b	a	c

Ranked Choice Winner: a

6	5	4	2
a	c	b	a
b	a	c	b
c	b	a	c

More-is-Less Paradox: Ranked Choice

6	5	4	2
a	c	b	b
b	a	c	a
c	b	a	c

Ranked Choice Winner: a

6	5	4	2
a	c	b	a
b	a	c	b
c	b	a	c

Ranked Choice Winner: c

More-is-Less Paradox: Ranked Choice

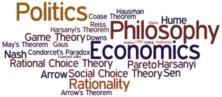
6	5	4	2
a	c	b	b
b	a	c	a
c	b	a	c

Ranked Choice Winner: a

6	5	4	2
a	c	b	a
b	a	c	b
c	b	a	c

Ranked Choice Winner: c

More on Monotonicity



Key idea: Unequivocal increase in support for a candidate should not result in that candidate going from being a winner to being a loser.

More on Monotonicity

Key idea: Unequivocal increase in support for a candidate should not result in that candidate going from being a winner to being a loser.

Monotonicity: if a candidate x is a winner given a preference profile $\mathbf{P}$, and $\mathbf{P}'$ is obtained from $\mathbf{P}$ by one voter moving x up in their ranking, then x should still be a winner given $\mathbf{P}'$.

More Principles

Pareto/Unanimity: In any profile **P**, if every voter ranks x strictly above y, then y is not a winner.

Every voting method we have studied satisfies Pareto.

More Principles

Condorcet: In any profile **P**, if x is a Condorcet winner, then x is the unique winner.

Condorcet Loser: In any profile **P**, if x is a Condorcet loser, then x is not a winner.

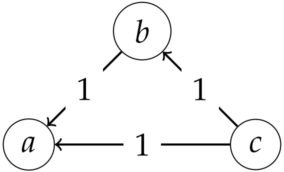
More Principles

Condorcet: In any profile **P**, if x is a Condorcet winner, then x is the unique winner.

Condorcet Loser: In any profile **P**, if x is a Condorcet loser, then x is not a winner.

Plurality violates both the Condorcet Winner and Condorcet Loser principles.

2	2	2	1
c	b	a	a
b	c	c	b
a	a	b	c



Plurality Winners: $\{a\}$
Condorcet Winner: c
Condorcet Loser: a

	Plurality	Borda	Ranked Choice	Coombs	Cope-land	Mini-max	Split Cycle
Anonymity	✓	✓	✓	✓	✓	✓	✓
Neutrality	✓	✓	✓	✓	✓	✓	✓
Pareto	✓	✓	✓	✓	✓	✓	✓

	Plurality	Borda	Ranked Choice	Coombs	Copeland	Minimax	Split Cycle
Anonymity	✓	✓	✓	✓	✓	✓	✓
Neutrality	✓	✓	✓	✓	✓	✓	✓
Pareto	✓	✓	✓	✓	✓	✓	✓
Condorcet Winner	—	—	—	—	✓	✓	✓
Condorcet Loser	—	✓	✓	✓	✓	—	✓

	Plurality	Borda	Ranked Choice	Coombs	Copeland	Minimax	Split Cycle
Anonymity	✓	✓	✓	✓	✓	✓	✓
Neutrality	✓	✓	✓	✓	✓	✓	✓
Pareto	✓	✓	✓	✓	✓	✓	✓
Condorcet Winner	—	—	—	—	✓	✓	✓
Condorcet Loser	—	✓	✓	✓	✓	—	✓
Monotonicity	✓	✓	—	—	✓	✓	✓