PHPE 400
Individual and Group Decision Making

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Majority Rule

Majority Rule: a is ranked above (below) b if more (fewer) voters rank a above b than b above a, otherwise a and b are tied.

When there are only two options, can we argue that majority rule is the “best” procedure?

May’s Theorem is a *proceduralist* justification of majority rule showing that Majority Rule is the unique group decision method satisfying two basic principles of fairness (Anonymity and Neutrality) and a basic principle ensuring that the outcome responds appropriately to the voters’ opinions (Weak Positive Responsiveness).

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We can also give an *epistemic* justification of majority rule showing that has a high probability of identifying the correct answer to a question.

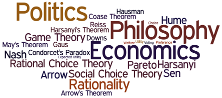
Epistemic Justification of Majority Rule

In many group decision making problems, one of the alternatives is the *correct* one. Which group decision making method is best for finding the "correct" alternative?

The Condorcet Jury Theorem

https://cjt-tutorial.streamlit.app/

Condorcet Jury Theorem



- $V = \{1, 2, \ldots, n\}$ is the set of experts.
- $\{0, 1\}$ is the set of outcomes.
- $\mathbf{x}$ be a random variable (called the **state**) whose values range over the two outcomes. We write $\mathbf{x} = 1$ when the outcome is 1 and $\mathbf{x} = 0$ when the outcome is 0.
- $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are random variables representing the votes for experts $1, 2, \ldots, n$. For each $i = 1, \ldots, n$, we write $\mathbf{v}_i = 1$ when expert i's vote is 1 and $\mathbf{v}_i = 0$ when expert i's vote is 0.
- R_i is the event that expert i votes correctly: it is the event that $\mathbf{v}_i$ coincides with $\mathbf{x}$ (i.e., $\mathbf{v}_i = 1$ and $\mathbf{x} = 1$ or $\mathbf{v}_i = 0$ and $\mathbf{x} = 0$).

Condorcet Jury Theorem

Independence: The correctness events $R_1, R_2, \ldots, R_n$ are independent.

Competence: The experts' competences $Pr(R_i)$ (i) exceeds $\frac{1}{2}$ and (ii) is the same for each voter i.

Condorcet Jury Theorem: Assume Independence and Competence. Then, as the group size increases, the probability of that the majority is correct (i) increases (growing reliability), and (ii) tends to one (infallibility).

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The Condorcet Jury Theorem is an *epistemic* justification of majority rule showing that under the assumption that the voters are *competent* in the sense that each voters has a greater than 50% chance of voting correctly and that the events that the voters are correct are independent, then the probability that the majority is correct increases to 1 as the size of the group increases.

What happens when there are more than 2 candidates?

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- Group decision problems often exhibit a *combinatorial structure*. For example, voting on a number of yes/no issues in a referendum, or voting on different interconnected issues, or selecting a committee from a set of candidates.
- As we have seen, there are many different reasonable voting methods that generalize Majority Rule for more than 2 candidates.

Multiple Elections Paradox

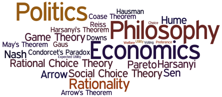
S. Brams, D. M. Kilgour, and W. Zwicker (1998). *The paradox of multiple elections*. Social Choice and Welfare, 15(2), pp. 211 - 236.

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

Multiple Elections Paradox



Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
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Outcome by majority vote

Proposition 1: N (7 - 6)

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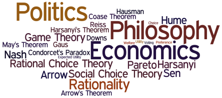


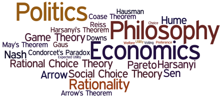
Voters are asked to give their opinion on three yes/no issues:

YYY	*YYN*	*YNY*	*YNN*	*NYY*	*NYN*	*NNY*	*NNN*
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: N (7 - 6)
Proposition 2: N (7 - 6)

Multiple Elections Paradox



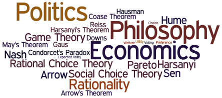
Voters are asked to give their opinion on three yes/no issues:

YYY	*YYN*	*YNY*	*YNN*	*NYY*	*NYN*	*NNY*	*NNN*
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: *N* (7 - 6)
Proposition 2: *N* (7 - 6)
Proposition 3: *N* (7 - 6)

Multiple Elections Paradox



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YYY	*YYN*	*YNY*	*YNN*	*NYY*	*NYN*	*NNY*	*NNN*
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: *N* (7 - 6)
Proposition 2: *N* (7 - 6)
Proposition 3: *N* (7 - 6)

But there is no support for NNN!

S. Brams, M. Kilgour and W. Zwicker (1997). *Voting on referenda: the separability problem and possible solutions*. Electoral Studies, 16(3), pp. 359 - 377.

D. Lacy and E. Niou (2000). *A problem with referenda*. Journal of Theoretical Politics 12(1), pp. 5 - 31.

J. Lang and L. Xia (2009). *Sequential composition of voting rules in multi-issue domains*. Mathematical Social Sciences 57(3), pp. 304 - 324.

L. Xia, V. Conitzer and J. Lang (2010). *Strategic Sequential Voting in Multi-Issue Domains and Multiple-Election Paradoxes*. In Proceedings of the Twelfth ACM Conference on Electronic Commerce (EC-11), pp. 179-188.

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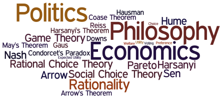

"Is a conflict between the proposition and combination winners necessarily bad? ... The paradox does not just highlight problems of aggregation and packaging, however, but strikes at the core of social choice—both what it means and how to uncover it. In our view, the paradox shows there may be a clash between two different meanings of social choice, leaving unsettled the best way to uncover what this elusive quantity is." (pg. 234).

S. Brams, D. M. Kilgour, and W. Zwicker. *The paradox of multiple elections*. Social Choice and Welfare, 15(2), pgs. 211 - 236, 1998.