PHPE 400
Individual and Group Decision Making

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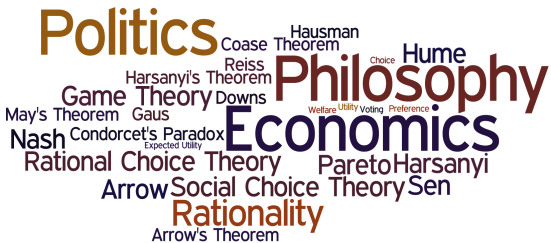

Principles of group decision making

- **Anonymity**: If voters swap their ballots, then the outcome is unaffected.
- **Neutrality**: If candidates are exchanged in every ranking, then the outcome changes accordingly.
- **Resoluteness**: Always elect a single winner.

Condorcet Triples and Resoluteness

n	n	n
a	b	c
b	c	a
c	a	b

n	n	n
a	c	b
c	b	a
b	a	c

Fact. In both profiles, any voting method satisfying anonymity and neutrality must select all candidates as winners

1	1	1
a	b	c
b	c	a
c	a	b

Consider $\mathbf{P} = (a\ b\ c, b\ c\ a, c\ a\ b)$ and suppose that $F(a\ b\ c, b\ c\ a, c\ a\ b) = \{a\}$

Suppose that $F($ a b c, b c a, c a b $) = \{$ a $\}$

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1. Swap a and b in everyone's rankings in the given profile. Then, by Neutrality:

$$F(b\ a\ c, a\ c\ b, c\ b\ a) = \{b\}$$

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2. Swap b and c in everyone's rankings in the profile from step 1. Then, by Neutrality:
$$F(c\ a\ b, a\ b\ c, b\ c\ a) = \{c\}$$

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2. Swap b and c in everyone's rankings in the profile from step 1. Then, by Neutrality:
$$F(c\ a\ b, a\ b\ c, b\ c\ a) = \{c\}$$

3. By Anonymity, the original profile and the profile in step 3 must have the same winners:
$$F(a\ b\ c, b\ c\ a, c\ a\ b) = F(c\ a\ b, a\ b\ c, b\ c\ a)$$

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$$F(a\ b\ c, b\ c\ a, c\ a\ b) = F(c\ a\ b, a\ b\ c, b\ c\ a)$$

4. 1 and 2 contradict 3 since
$F(a\ b\ c, b\ c\ a, c\ a\ b) = \{a\} \neq \{c\} = F(c\ a\ b, a\ b\ c, b\ c\ a)$.

So, tie-breaking cannot be built-in to a voting method: there is no voting method that satisfies Anonymity, Neutrality and always elects a single winner.

Dominance Principles

Unanimity

Pareto/Unanimity: In any profile **P**, if every voter ranks x strictly above y, then y is not a winner.

Every voting method we have studied satisfies Pareto.

Condorcet Winner/Loser

Condorcet: In any profile **P**, if x is a Condorcet winner, then x is the unique winner.

Condorcet Loser: In any profile **P**, if x is a Condorcet loser, then x is not a winner.

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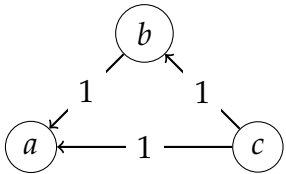
Plurality violates both the Condorcet Winner and Condorcet Loser principles.

2	2	2	1
c	b	a	a
b	c	c	b
a	a	b	c



Plurality Winners: $\{a\}$
Condorcet Winner: c
Condorcet Loser: a

	Plurality	Borda	Instant Runoff	Coombs	Cope-land	Mini-max	MWSL
Anonymity	✓	✓	✓	✓	✓	✓	✓
Neutrality	✓	✓	✓	✓	✓	✓	✓
Pareto	✓	✓	✓	✓	✓	✓	✓

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Pareto	✓	✓	✓	✓	✓	✓	✓
Condorcet Winner	–	–	–	–	✓	✓	✓
Condorcet Loser	–	✓	✓	✓	✓	–	✓

Recall Weak Positive Responsiveness

- F satisfies **weak positive responsiveness** if for any profiles $\mathbf{P}$ and $\mathbf{P}'$, if
  1. $a \in F(\mathbf{P})$ (a is a winner in $\mathbf{P}$ according to F) and
  2. $\mathbf{P}'$ is obtained from $\mathbf{P}$ by one voter who ranked a uniquely last in $\mathbf{P}$ switching to ranking a uniquely first in $\mathbf{P}'$,

  then $F(\mathbf{P}') = \{a\}$ (a is the **unique** winner in $\mathbf{P}'$ according to F).

Monotonicity

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More-is-Less Paradox: If a candidate c is elected under a given a profile of rankings of the competing candidates, it is possible that, *ceteris paribus*, c may not be elected if some voter(s) raise c in their rankings.

P. Fishburn and S. Brams. *Paradoxes of Preferential Voting*. Mathematics Magazine (1983).

More-is-Less Paradox: Instant Runoff Voting

6	5	4	2
a	c	b	b
b	a	c	a
c	b	a	c

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a	c	b	a
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c	b	a	c

More-is-Less Paradox: Instant Runoff Voting

6	5	4	2
a	*c*	*b*	*b*
b	*a*	*c*	*a*
c	*b*	*a*	*c*

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Instant Runoff Winner: *a*

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Instant Runoff Winner: *a*

6	5	4	2
a	*c*	*b*	*a*
b	*a*	*c*	*b*
c	*b*	*a*	*c*

Instant Runoff Winner: *c*

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b	*a*	*c*	*a*
c	*b*	*a*	*c*

Instant Runoff Winner: *a*

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b	*a*	*c*	*b*
c	*b*	*a*	*c*

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Condorcet Winner	–	–	–	–	✓	✓	✓
Condorcet Loser	–	✓	✓	✓	✓	–	✓
Monotonicity	✓	✓	–	–	✓	✓	✓

The Spoiler Problem

$2,912,790$	$2,912,253$	$97,488$
Bush	Gore	Nader
Gore	Nader	Gore
Nader	Bush	Bush

Nader *spoiled* the election for Gore.

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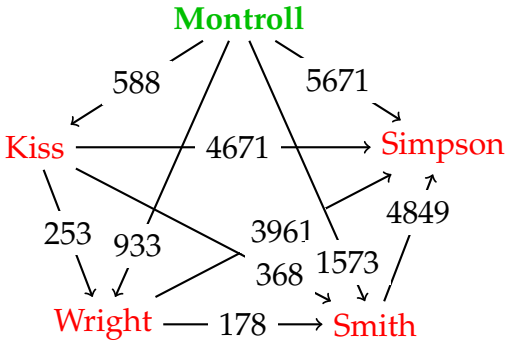

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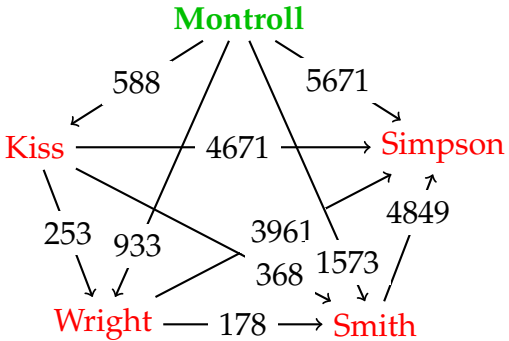

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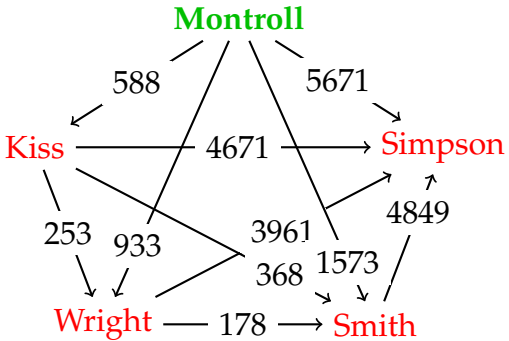

Montroll was the **Condorcet winner**. IRV was repealed in 2010.

The Spoiler Problem

37	29	34
d	d	p
p	p	d

Instant Runoff winner: d

37	29	34
r	d	p
d	p	d
p	r	r

Instant Runoff winner: p

r spoils the election for d: A majority prefers d to r, but the addition of r knocks d out of the winning set.

(See www.electionscience.org/library/the-spoiler-effect/)

Examples of spoiler effects

- **2000 Florida Presidential Election** (Plurality):

  Gore would have won had the election not included Nader, whom Gore (plausibly) beat head-to-head. But with Nader included, Bush won.

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  Montroll would have won had the election not included Wright, whom Montroll beat head-to-head. But with Wright included, Kiss won.

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  Gore would have won had the election not included Nader, whom Gore (plausibly) beat head-to-head. But with Nader included, Bush won.

- **2007 Burlington Mayoral Election** (Instant Runoff):

  Montroll would have won had the election not included Wright, whom Montroll beat head-to-head. But with Wright included, Kiss won.

- **2022 Special Election for U.S. Rep. in Alaska** (Instant Runoff):

  Begich would have won had the election not included Palin, whom Begich beat head-to-head. But with Palin included, Peltola won.

Immunity to Spoilers

A voting method satisfies **Immunity to Spoilers** if the following *can't happen*:

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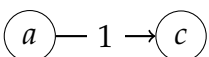
This criterion rules out all the spoiler effects we've discussed.

But do any useable voting methods satisfy it—or is it too good to be true?

Broda

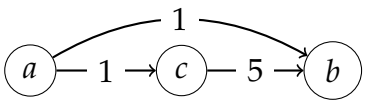
Borda violates Immunity to Spoilers:

2	3
c	a
a	c



Borda winner: a

2	3
c	a
b	c
a	b



Borda winner: c

Broda

Borda violates Immunity to Spoilers:

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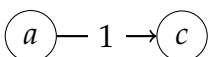
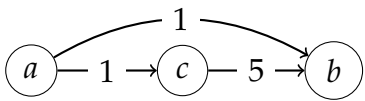
c	a
a	c



Borda winner: a

2	3
c	a
b	c
a	b



Borda winner: c

Let **P** be the election on the right.

- a is a Borda winner without b in the election **P**
- a is majority preferred to b in **P**
- a and b both lose in **P** according to Borda

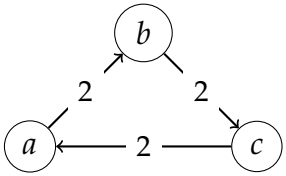
The only voting methods that you have seen so far that satisfy Immunity to Spoilers is Minimax and Maximum Wins, Smallest Loss.

	Plurality	Borda	Instant Runoff	Coombs	Cope-land	Mini-max	MWSL
Anonymity	✓	✓	✓	✓	✓	✓	✓
Neutrality	✓	✓	✓	✓	✓	✓	✓
Pareto	✓	✓	✓	✓	✓	✓	✓
Condorcet Winner	–	–	–	–	✓	✓	✓
Condorcet Loser	–	✓	✓	✓	✓	–	✓
Monotonicity	✓	✓	–	–	✓	✓	✓
Immunity to Spoilers	–	–	–	–	–	✓	✓

Multiple-Districts Paradox

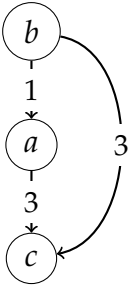
Multiple-Districts: If a candidate wins in each district, then that candidate should also win when the districts are merged.

Multiple-Districts Paradox



2	2	2
a	b	c
b	c	a
c	a	b

1	2
a	b
b	a
c	c



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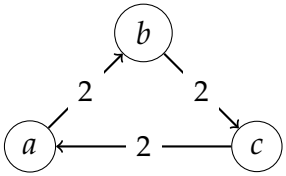
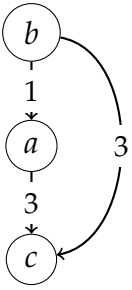
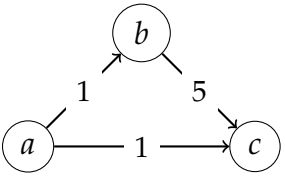


2	2	2
a	b	c
b	c	a
c	a	b

1	2
a	b
b	a
c	c





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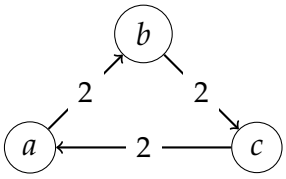
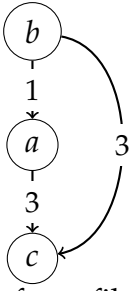
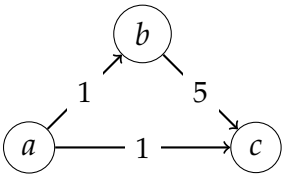


2	2	2
a	b	c
b	c	a
c	a	b

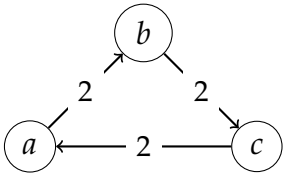
1	2
a	b
b	a
c	c





- $\{a, b, c\}$ are the winners in the left profile (assuming Anonymity and Neutrality)
- b is the Condorcet winner in the right profile
- a is the Condorcet winner in the combined profiles

Multiple-Districts Paradox



2	2	2
a	b	c
b	c	a
c	a	b

1	2
a	b
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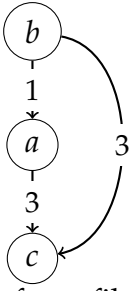
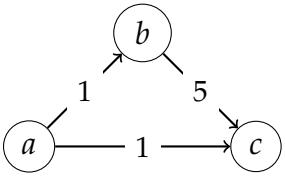
c	c





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So, any Condorcet consistent voting method violates the Multiple-Districts Property.

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Anonymity	✓	✓	✓	✓	✓	✓	✓
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Pareto	✓	✓	✓	✓	✓	✓	✓
Condorcet Winner	–	–	–	–	✓	✓	✓
Condorcet Loser	–	✓	✓	✓	✓	–	✓
Monotonicity	✓	✓	–	–	✓	✓	✓
Immunity to Spoilers	–	–	–	–	–	✓	✓
Multiple Districts	✓	✓	–	–	–	–	–