PHPE 400
Individual and Group Decision Making

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Justifying Majority Rule

May's Theorem is a **proceduralist** justification of majority rule showing that Majority Rule is the unique group decision method satisfying two basic principles of fairness (Anonymity and Neutrality) and a basic principle ensuring that the outcome responds appropriately to the voters' opinions (Weak Positive Responsiveness).

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The Condorcet Jury Theorem is an **epistemic** justification of majority rule showing that under the assumption that the voters are *competent* in the sense that each voters has a greater than 50% chance of voting correctly and that the events that the voters are correct are independent, then the probability that the majority is correct increases to 1 as the size of the group increases.

Beyond Two Candidates

With 2 candidates, Majority Rule is uniquely justified (May's Theorem, Condorcet Jury Theorem)

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With more than 2 candidates, there are two problems:

1. With multiple issues/propositions, voting on each issue separately can contradict voting on packages of issues
2. There is no single extension of majority rule to three or more candidates (e.g., Plurality, Borda, Instant Runoff, etc.).

Multiple Elections Paradox

S. Brams, D. M. Kilgour, and W. Zwicker (1998). *The paradox of multiple elections*. Social Choice and Welfare, 15(2), pp. 211 - 236.

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

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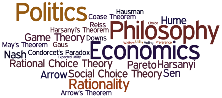
<u>Outcome by majority vote</u>

Proposition 1: N (7 - 6)

Multiple Elections Paradox



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YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: N (7 - 6)
Proposition 2: N (7 - 6)

Multiple Elections Paradox

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YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: N (7 - 6)
Proposition 2: N (7 - 6)
Proposition 3: N (7 - 6)

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	*YYN*	*YNY*	*YNN*	*NYY*	*NYN*	*NNY*	*NNN*
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: N (7 - 6)
Proposition 2: N (7 - 6)
Proposition 3: N (7 - 6)

But there is no support for NNN!

S. Brams, M. Kilgour and W. Zwicker (1997). *Voting on referenda: the separability problem and possible solutions*. Electoral Studies, 16(3), pp. 359 - 377.

D. Lacy and E. Niou (2000). *A problem with referenda*. Journal of Theoretical Politics 12(1), pp. 5 - 31.

J. Lang and L. Xia (2009). *Sequential composition of voting rules in multi-issue domains*. Mathematical Social Sciences 57(3), pp. 304 - 324.

L. Xia, V. Conitzer and J. Lang (2010). *Strategic Sequential Voting in Multi-Issue Domains and Multiple-Election Paradoxes*. In Proceedings of the Twelfth ACM Conference on Electronic Commerce (EC-11), pp. 179-188.

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"Is a conflict between the proposition and combination winners necessarily bad? ... The paradox does not just highlight problems of aggregation and packaging, however, but strikes at the core of social choice—both what it means and how to uncover it. In our view, the paradox shows there may be a clash between two different meanings of social choice, leaving unsettled the best way to uncover what this elusive quantity is." (pg. 234).

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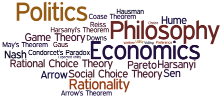
Judgement Aggregation Paradoxes



Kornhauser and Sager. *Unpacking the court*. Yale Law Journal, 1986.

P. Mongin. *The doctrinal paradox, the discursive dilemma, and logical aggregation theory*. Theory and Decision, 73(3), pp 315 - 355, 2012.

C. List and P. Pettit. *Aggregating sets of judgments: An impossibility result*. Economics and Philosophy 18, pp. 89 - 110, 2002.

Judgement Aggregation Paradox

Should we hire the candidate?

- Is the candidate good at research (r)?
- Is the candidate good at teaching (t)?
- We should hire the candidate if and only if the candidate is good at research and teaching. ($r \wedge t$)

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

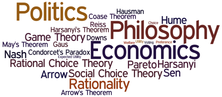
	r	t		h
Voter 1				
Voter 2				
Voter 3				
Group				

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

	r	t		h
Voter 1	Yes	Yes		
Voter 2	Yes	No		
Voter 3	No	Yes		
Group	Yes	Yes		

Judgement Aggregation Paradox



Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes		
Voter 2	Yes	No		
Voter 3	No	Yes		
Group	Yes	Yes	Yes	Yes

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes	Yes	Yes
Voter 2	Yes	No	Yes	No
Voter 3	No	Yes	Yes	No
Group				No

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes	Yes	Yes
Voter 2	Yes	No	Yes	No
Voter 3	No	Yes	Yes	No
Group	Yes	Yes	Yes	Y/N

Beyond Two Candidates

With 2 candidates, Majority Rule is uniquely justified (May's Theorem, Condorcet Jury Theorem)

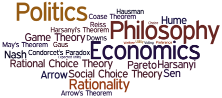
With more than 2 candidates, there are two problems:

✓ With multiple issues/propositions, voting on each issue separately can contradict voting on packages of issues

2. There is no single extension of majority rule to three or more candidates (e.g., Plurality, Borda, Instant Runoff, etc.).

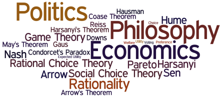
   What additional principles should we use to distinguish between the different voting methods?

Principles of group decision making



- **Anonymity**: If voters swap their ballots, then the outcome is unaffected.
- **Neutrality**: If candidates are exchanged in every ranking, then the outcome changes accordingly.
- **Resoluteness**: Always elect a single winner.

Condorcet Triples and Resoluteness



n	n	n
a	b	c
b	c	a
c	a	b

n	n	n
a	c	b
c	b	a
b	a	c

Fact. In both profiles, any voting method satisfying anonymity and neutrality must select all candidates as winners

1	1	1
a	b	c
b	c	a
c	a	b

Consider $\mathbf{P} = (a\ b\ c, b\ c\ a, c\ a\ b)$ and suppose that $F(a\ b\ c, b\ c\ a, c\ a\ b) = \{a\}$

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1. Swap a and b in everyone's rankings in the given profile. Then, by Neutrality:

$$F(b\ a\ c, a\ c\ b, c\ b\ a) = \{b\}$$

Suppose that $F(\boxed{a}\ \boxed{b}\ \boxed{c}, \boxed{b}\ \boxed{c}\ \boxed{a}, \boxed{c}\ \boxed{a}\ \boxed{b}) = \{\boxed{a}\}$

1. Swap a and b in everyone's rankings in the given profile. Then, by Neutrality:

$$F(\boxed{b}\ \boxed{a}\ c, \boxed{a}\ c\ \boxed{b}, c\ \boxed{b}\ \boxed{a}) = \{\boxed{b}\}$$

2. Swap b and c in everyone's rankings in the profile from step 1. Then, by Neutrality:

$$F(\boxed{c}\ a\ \boxed{b}, a\ \boxed{b}\ \boxed{c}, \boxed{b}\ \boxed{c}\ a) = \{\boxed{c}\}$$

Suppose that $F(a\ b\ c, b\ c\ a, c\ a\ b) = \{a\}$

1. Swap a and b in everyone's rankings in the given profile. Then, by Neutrality:
$$F(b\ a\ c, a\ c\ b, c\ b\ a) = \{b\}$$

2. Swap b and c in everyone's rankings in the profile from step 1. Then, by Neutrality:
$$F(c\ a\ b, a\ b\ c, b\ c\ a) = \{c\}$$

3. By Anonymity, the original profile and the profile in step 3 must have the same winners:

$$F(a\ b\ c, b\ c\ a, c\ a\ b) = F(c\ a\ b, a\ b\ c, b\ c\ a)$$

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4. 1 and 2 contradict 3 since
$F(a\ b\ c, b\ c\ a, c\ a\ b) = \{a\} \neq \{c\} = F(c\ a\ b, a\ b\ c, b\ c\ a)$.

So, tie-breaking cannot be built-in to a voting method: there is no voting method that satisfies Anonymity, Neutrality and always elects a single winner.