PHPE 400
Individual and Group Decision Making

Eric Pacuit
University of Maryland
pacuit.org

Majority Rule

When there are only **two** candidates a and b, then all voting methods give the same results

Majority Rule

When there are only **two** candidates a and b, then all voting methods give the same results

Majority Rule: a is ranked above (below) b if more (fewer) voters rank a above b than b above a, otherwise a and b are tied.

When there are only two options, can we argue that majority rule is the "best" procedure?

Democracy: The decisions made by a group must be appropriately responsive to the expressed wishes of the members of that group.

Political equality: Each group member must have an equal (chance of) influence over the group's decisions.

Majority rule: The option that gets the most votes should be the group decision.

B. Saunders (2010). *Democracy, Political Equality, and Majority Rule*. Ethics, 121(1), pp. 148-177.

Lottery Voting

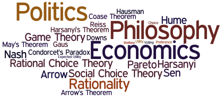
Lottery voting: each person casts a vote for their favored option but, rather than the option with most votes automatically winning, a single vote is randomly selected and that one determines the outcome.

Lottery Voting

Lottery voting: each person casts a vote for their favored option but, rather than the option with most votes automatically winning, a single vote is randomly selected and that one determines the outcome.

- This procedure is democratic, since all members of the community have a chance to influence outcomes

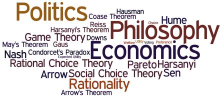
Lottery Voting



Lottery voting: each person casts a vote for their favored option but, rather than the option with most votes automatically winning, a single vote is randomly selected and that one determines the outcome.

- This procedure is democratic, since all members of the community have a chance to influence outcomes
- It is egalitarian, since all have an equal chance of being picked. It gives each voter an equal chance of being decisive, but voters do not have equal chances of getting their way—rather, the chance of each option winning is proportional to the number of votes it obtains.

Lottery Voting



Lottery voting: each person casts a vote for their favored option but, rather than the option with most votes automatically winning, a single vote is randomly selected and that one determines the outcome.

- This procedure is democratic, since all members of the community have a chance to influence outcomes
- It is egalitarian, since all have an equal chance of being picked. It gives each voter an equal chance of being decisive, but voters do not have equal chances of getting their way—rather, the chance of each option winning is proportional to the number of votes it obtains.
- It is not majority rule, since the vote of someone in the minority may be picked.

Lottery Voting

This shows that democracy and political equality do not conceptually require majority rule.

(Saunders argues that there are no clearly decisive general reasons to prefer majority rule to lottery voting in all cases.)

What justifies majority rule?

Minority vs. Majority: If a minority could prevail over the majority, those who were in favor of a proposition would vote against it, or would abstain from voting in order to insure a majority to their side of the question. Also, there would be no inducement to discuss a question, if, by converting a person to our opinion, you did not strengthen your side when the votes came to be counted.

M. Risse (2004). *Arguing for majority rule*. Journal of Political Philosophy 12 (1), pp. 41 - 64.

What justifies majority rule?

Respect: Majority rule is a good way of expressing respect for people in the circumstances of politics, that is, in circumstances in which in spite of remaining differences (even after deliberation) a common view needs to be found. Majority rule allows each person to remain faithful to their conviction, but still to accept that a group decision needs to be made.

M. Risse (2004). *Arguing for majority rule.* Journal of Political Philosophy 12 (1), pp. 41 - 64.

Justifying Majority Rule

When there are only two options, can we argue that majority rule is the "best" procedure?

Setting aside the possibility of using lotteries, May's Theorem is a proceduralist justification of majority rule showing that it is the unique procedure satisfying normative principles of group decision making.

K. May (1952). *A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision*. Econometrica, Vol. 20.

May's Theorem: Details

Voters: $V = \{1, 2, 3, \ldots, n\}$ is the set of n voters.

Candidates: $X = \{a, b\}$ is set of candidates.

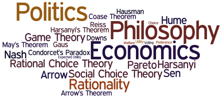
Suppose that voters can submit one of 3 rankings:

1. $a\ P\ b$: a is ranked above b ("vote for a")
2. $a\ I\ b$: a and b are tied ("vote for a and b")
3. $b\ P\ a$: b is ranked above a ("vote for b")

Note that $a\ I\ b$ and $b\ I\ a$ is the same ballot since indifference is symmetric.

Let $O(X)$ be the set of 3 rankings on X.

May's Theorem: Details



The set of **profiles** is $O(X)^V$, where a profile assigns to each voter one of the three rankings from $O(X)$.

Given a profile $\mathbf{P} \in O(X)^V$ and a voter $i \in V$, we write $\mathbf{P}_i$ for the ranking of voter i.

E.g., suppose that $V = \{1, 2, 3, 4\}$ and consider the profile

$$\mathbf{P} = (a\ P\ b, a\ I\ b, b\ P\ a, a\ P\ b)$$

Then, $\mathbf{P}_2$ is the ranking $a\ I\ b$ (voter 2 is indifferent between a and b).

May's Theorem: Details

Social Choice Function: $F : O(X)^V \to \wp(X)$.

Where for all profiles $\mathbf{P}$ from $O(X)^V$, $F(\mathbf{P})$ is the set of winners.

We assume that for all profile $\mathbf{P}$, $F(\mathbf{P}) \neq \varnothing$ (so there is always at least one winner).

May's Theorem: Details

Social Choice Function: $F : O(X)^V \to \wp(X)$.

Examples:

- Majority rule: The winner is the candidate with the most votes, otherwise the candidates are tied
- Quota rule: The winner is the candidate with more than q% of the vote (e.g., more than 2/3 of the vote), otherwise the candidates are tied.
- Unanimity rule: A candidate wins is *all voters* vote for that candidate, otherwise the candidates are tied.

May’s Theorem: Details

Social Choice Function: $F : O(X)^V \to \wp(X)$.

Examples:

- Minority rule: The winner is the candidate with the fewest votes, otherwise the candidates are tied.
- Majority rule with status quo: The winner is the candidate with the most votes, and if there is a tie candidate a wins.
- Candidate a always wins.
- The winner is whoever voter 1 voted for.
- The candidates are always tied.

May’s Theorem: Details

$$F_{Maj}(\mathbf{P}) = \begin{cases} \{a\} & \text{if more voters rank } a \text{ above } b \text{ than } b \text{ above } a \\ \{a, b\} & \text{if the same number of voters rank } a \text{ above } b \text{ as } b \text{ above } a \\ \{b\} & \text{if more voters rank } b \text{ above } a \text{ than } a \text{ above } b \end{cases}$$

May's Theorem: Details

$$F_{Maj}(\mathbf{P}) = \begin{cases} \{a\} & \text{if } Margin_{\mathbf{R}}(a,b) > 0 \\ \{a,b\} & \text{if } Margin_{\mathbf{R}}(a,b) = 0 \\ \{b\} & \text{if } Margin_{\mathbf{R}}(b,a) > 0 \end{cases}$$

Anonymity and Neutrality

- F satisfies **anonymity**: permuting the voters does not change the set of winners.
- F satisfies **neutrality**: permuting the candidates results in a winning set that is permuted in the same way.

Anonymity and Neutrality

- F satisfies **anonymity**: permuting the voters does not change the set of winners.
- F satisfies **neutrality**: permuting the candidates results in a winning set that is permuted in the same way.

$\Longrightarrow$ in 2-candidate profiles, if the same number of voters rank $\boldsymbol{a}$ above $\boldsymbol{b}$ as $\boldsymbol{b}$ above $\boldsymbol{a}$, then $\boldsymbol{a} \in F(\mathbf{P})$ if, and only if, $\boldsymbol{b} \in F(\mathbf{P})$

($\boldsymbol{a}$ wins according to F if and only if $\boldsymbol{b}$ wins according to F).

Weak Positive Responsiveness

- F satisfies **weak positive responsiveness** if for any profiles $\mathbf{P}$ and $\mathbf{P}'$, if

  1. $a \in F(\mathbf{P})$ (a is a winner in $\mathbf{P}$ according to F) and
  2. $\mathbf{P}'$ is obtained from $\mathbf{P}$ by one voter who ranked a uniquely last in $\mathbf{P}$ switching to ranking a uniquely first in $\mathbf{P}'$ (while all other voters' rankings are unchanged),

  then $F(\mathbf{P}') = \{a\}$ (a is the **unique** winner in $\mathbf{P}'$ according to F).

Profile	Voter 1	Always a	Minority	Consensus	Majority
$(a\ P\ b, a\ P\ b)$	a	a	b	a	a
$(a\ P\ b, a\ I\ b)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a)$	a	a	a, b	a, b	a, b
$(a\ I\ b, a\ P\ b)$	a, b	a	b	a, b	a
$(a\ I\ b, a\ I\ b)$	a, b	a	a, b	a, b	a, b
$(a\ I\ b, b\ P\ a)$	a, b	a	a	a, b	b
$(b\ P\ a, a\ P\ b)$	b	a	a, b	a, b	a, b
$(b\ P\ a, a\ I\ b)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a)$	b	a	a	b	b

Profile	Voter 1	Always a	Minority	Consensus	Majority
$(a\ P\ b, a\ P\ b)$	a	a	b	a	a
$(a\ P\ b, a\ I\ b)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a)$	a	a	a, b	a, b	a, b
$(a\ I\ b, a\ P\ b)$	a, b	a	b	a, b	a
$(a\ I\ b, a\ I\ b)$	a, b	a	a, b	a, b	a, b
$(a\ I\ b, b\ P\ a)$	a, b	a	a	a, b	b
$(b\ P\ a, a\ P\ b)$	b	a	a, b	a, b	a, b
$(b\ P\ a, a\ I\ b)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a)$	b	a	a	b	b

	Anonymity	Neutrality	Weak Positive Responsiveness
Voter 1			
Always a			
Minority			
Consensus			
Majority			

Profile	Voter 1	Always a	Minority	Consensus	Majority
$(a\ P\ b, a\ P\ b)$	a	a	b	a	a
$(a\ P\ b, a\ I\ b)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a)$	a	a	a, b	a, b	a, b
$(a\ I\ b, a\ P\ b)$	a, b	a	b	a, b	a
$(a\ I\ b, a\ I\ b)$	a, b	a	a, b	a, b	a, b
$(a\ I\ b, b\ P\ a)$	a, b	a	a	a, b	b
$(b\ P\ a, a\ P\ b)$	b	a	a, b	a, b	a, b
$(b\ P\ a, a\ I\ b)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a)$	b	a	a	b	b

	Anonymity	Neutrality	Weak Positive Responsiveness
Voter 1	✗		
Always a	✓		
Minority	✓		
Consensus	✓		
Majority	✓		

Profile	Voter 1	Always a	Minority	Consensus	Majority
$(a\ P\ b, a\ P\ b)$	a	a	b	a	a
$(a\ P\ b, a\ I\ b)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a)$	a	a	a, b	a, b	a, b
$(a\ I\ b, a\ P\ b)$	a, b	a	b	a, b	a
$(a\ I\ b, a\ I\ b)$	a, b	a	a, b	a, b	a, b
$(a\ I\ b, b\ P\ a)$	a, b	a	a	a, b	b
$(b\ P\ a, a\ P\ b)$	b	a	a, b	a, b	a, b
$(b\ P\ a, a\ I\ b)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a)$	b	a	a	b	b

	Anonymity	Neutrality	Weak Positive Responsiveness
Voter 1	✗	✓	
Always a	✓	✗	
Minority	✓	✓	
Consensus	✓	✓	
Majority	✓	✓	

	Profile	Voter 1	Always a	Minority	Consensus	Majority
$\mathbf{P}'$	$(a\ P\ b, a\ P\ b)$	a	a	b	a	a
	$(a\ P\ b, a\ I\ b)$	a	a	b	a, b	a
$\mathbf{P}$	$(a\ P\ b, b\ P\ a)$	a	a	a, b	a, b	a, b
$\mathbf{P}'$	$(a\ I\ b, a\ P\ b)$	a, b	a	b	a, b	a
	$(a\ I\ b, a\ I\ b)$	a, b	a	a, b	a, b	a, b
$\mathbf{P}$	$(a\ I\ b, b\ P\ a)$	a, b	a	a	a, b	b
$\mathbf{P}'$	$(b\ P\ a, a\ P\ b)$	b	a	a, b	a, b	a, b
	$(b\ P\ a, a\ I\ b)$	b	a	a	a, b	b
$\mathbf{P}$	$(b\ P\ a, b\ P\ a)$	b	a	a	b	b

	Anonymity	Neutrality	Weak Positive Responsiveness
Voter 1	✗	✓	✗
Always a	✓	✗	✓
Minority	✓	✓	✗
Consensus	✓	✓	✗
Majority	✓	✓	✓

	Anonymity	Neutrality	Weak Positive Responsiveness
Voter 1	✗	✓	✗
Always *a*	✓	✗	✓
Minority	✓	✓	✗
Consensus	✓	✓	✗
Majority	✓	✓	✓

May’s Theorem

Theorem (May 1952)

Let F be a voting method on the domain of two-alternative profiles. Then the following are equivalent:

1. F satisfies anonymity, neutrality, and weak positive responsiveness;
2. F is majority voting.

Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness.
Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$?

Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness.
Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$? No!

Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness. Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$? No!

Suppose that $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$

By Neutrality, $F(b\ P\ a, b\ P\ a, a\ P\ b) = \{a\}$

Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness.
Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$? No!

Suppose that $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$

By Neutrality, $F(b\ P\ a, b\ P\ a, a\ P\ b) = \{a\}$

By Anonymity, $F(a\ P\ b, b\ P\ a, b\ P\ a) = \{a\}$

Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness. Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$? No!

Suppose that $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$

By Neutrality, $F(b\ P\ a, b\ P\ a, a\ P\ b) = \{a\}$

By Anonymity, $F(a\ P\ b, b\ P\ a, b\ P\ a) = \{a\}$

By Weak Positive Responsiveness, $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a\}$

Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness. Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$? No!

Suppose that $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$

By Neutrality, $F(b\ P\ a, b\ P\ a, a\ P\ b) = \{a\}$

By Anonymity, $F(a\ P\ b, b\ P\ a, b\ P\ a) = \{a\}$

By Weak Positive Responsiveness, $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a\}$

Contradiction: Since F is a function, we can't have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$ and $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a\}$

Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness. Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a, b\}$? No!

Suppose that $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a, b\}$

By Neutrality, $F(b\ P\ a, b\ P\ a, a\ P\ b) = \{a, b\}$

By Anonymity, $F(a\ P\ b, b\ P\ a, b\ P\ a) = \{a, b\}$

By Weak Positive Responsiveness, $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a\}$

Contradiction: Since F is a function, we can't have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a, b\}$ and $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a\}$

May’s Theorem is a *proceduralist* justification of majority rule showing that Majority Rule is the unique group decision method satisfying two basic principles of fairness (Anonymity and Neutrality) and a basic principle ensuring that the outcome responds appropriately to the voters’ opinions (Weak Positive Responsiveness).

May's Theorem is a *proceduralist* justification of majority rule showing that Majority Rule is the unique group decision method satisfying two basic principles of fairness (Anonymity and Neutrality) and a basic principle ensuring that the outcome responds appropriately to the voters' opinions (Weak Positive Responsiveness).

We can also give an *epistemic* justification of majority rule showing that has a high probability of identifying the correct answer to a question.

Epistemic Justification of Majority Rule

In many group decision making problems, one of the alternatives is the *correct* one. Which group decision making method is best for finding the “correct” alternative?

The Condorcet Jury Theorem

https://cjt-tutorial.streamlit.app/

Condorcet Jury Theorem

- $V = \{1, 2, \ldots, n\}$ is the set of experts.
- $\{0, 1\}$ is the set of outcomes.
- $\mathbf{x}$ be a random variable (called the **state**) whose values range over the two outcomes. We write $\mathbf{x} = 1$ when the outcome is 1 and $\mathbf{x} = 0$ when the outcome is 0.
- $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are random variables representing the votes for experts $1, 2, \ldots, n$. For each $i = 1, \ldots, n$, we write $\mathbf{v}_i = 1$ when expert i's vote is 1 and $\mathbf{v}_i = 0$ when expert i's vote is 0.
- R_i is the event that expert i votes correctly: it is the event that $\mathbf{v}_i$ coincides with $\mathbf{x}$ (i.e., $\mathbf{v}_i = 1$ and $\mathbf{x} = 1$ or $\mathbf{v}_i = 0$ and $\mathbf{x} = 0$).

Condorcet Jury Theorem

Independence: The correctness events $R_1, R_2, \ldots, R_n$ are independent.

Competence: The experts' competences $Pr(R_i)$ (i) exceeds $\frac{1}{2}$ and (ii) is the same for each voter i.

Condorcet Jury Theorem: Assume Independence and Competence. Then, as the group size increases, the probability of that the majority is correct (i) increases (growing reliability), and (ii) tends to one (infallibility).