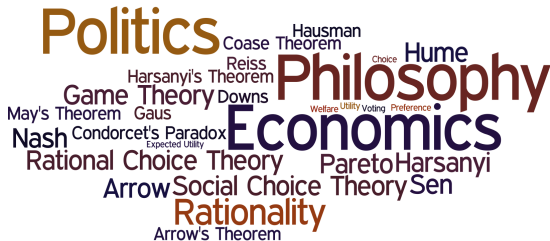


# PHPE 400

## Individual and Group Decision Making

Eric Pacuit  
University of Maryland  
[pacuit.org](http://pacuit.org)



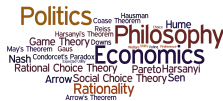
# Condorcet consistent voting methods



The **Condorcet winner** in a profile  $\mathbf{P}$  is a candidate  $x$  such that for all other candidates  $y$ ,  $\text{Margin}_{\mathbf{P}}(x, y) > 0$ .

A voting method is **Condorcet consistent**, if for all  $\mathbf{P}$ , if  $x$  is a Condorcet winner in  $\mathbf{P}$ , then  $x$  is the unique winner according to the voting method.

# Condorcet consistent voting methods

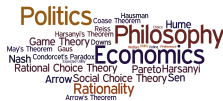


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We will study 3 Condorcet consistent voting methods: Copeland, Minimax, and Split Cycle.

# Condorcet consistent voting methods



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We will study 3 Condorcet consistent voting methods: Copeland, Minimax, and Split Cycle.

The Condorcet voting method Nanson was used in Marquette, Michigan, in the 1920s (Hoag and Hallett 1926, p. 491). To my knowledge, there are no cities using Condorcet consistent voting methods, but see the Condorcet Canada Initiative at <https://condorcet.ca>.

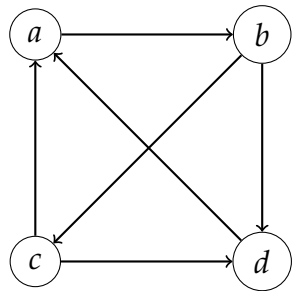
# Copeland



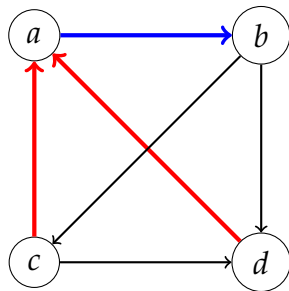
Say that the **win-loss record** for a candidate  $x$  is the number of candidates that  $x$  is majority preferred to minus the number of candidates that is majority preferred to  $y$ .

Then, any candidate with the largest win-loss record is a Copeland winner.

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

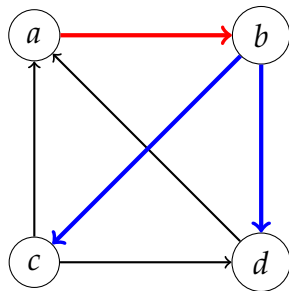


7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>



Win-loss record for *a*: 1 - 2 = -1

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

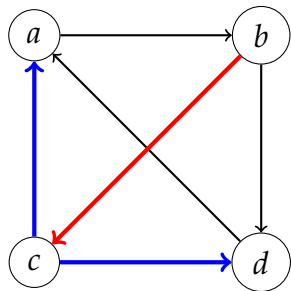


Win-loss record for *a*:  $1 - 2 = -1$

Win-loss record for *b*:  $2 - 1 = 1$



7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

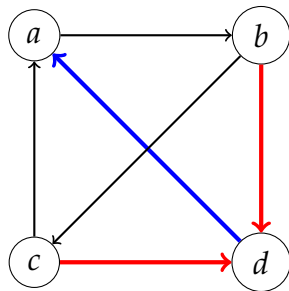


Win-loss record for *a*:  $1 - 2 = -1$

Win-loss record for *b*:  $2 - 1 = 1$

Win-loss record for *c*:  $2 - 1 = 1$

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>



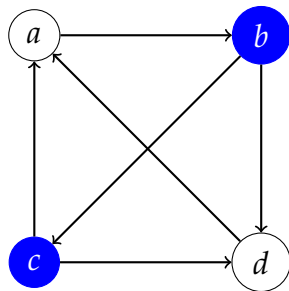
Win-loss record for *a*:  $1 - 2 = -1$

Win-loss record for *b*:  $2 - 1 = 1$

Win-loss record for *c*:  $2 - 1 = 1$

Win-loss record for *d*:  $1 - 2 = -1$

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>



Win-loss record for *a*:  $1 - 2 = -1$

Win-loss record for *b*:  $2 - 1 = 1$

Win-loss record for *c*:  $2 - 1 = 1$

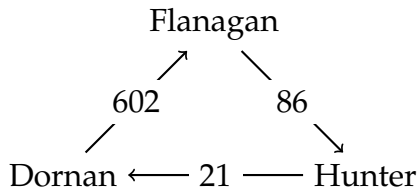
Win-loss record for *d*:  $1 - 2 = -1$

*c* and *b* are the Copeland winners.

# 2007 Glasgow City Council

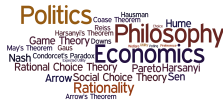


The top three candidates were in a **majority cycle**:

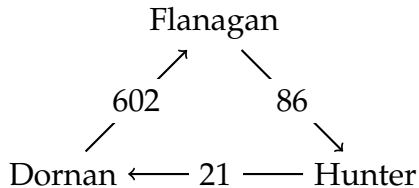


All candidates are tied according to Copeland (each candidate's win-loss record is 0).

# 2007 Glasgow City Council



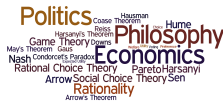
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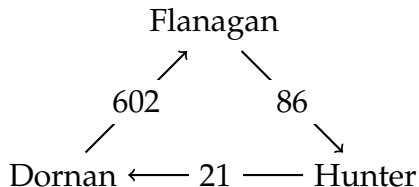
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Yet if we have to pick a single winner, and if we base our choice on the pairwise comparisons, it seems clear who the winner should be. . . .

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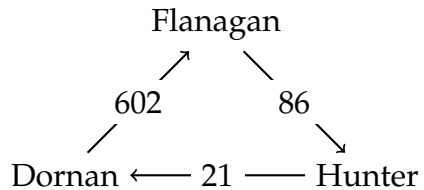
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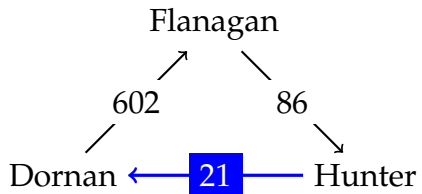
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Yet if we have to pick a single winner, and if we base our choice on the pairwise comparisons, it seems clear who the winner should be. . . .  
It's Dornan.

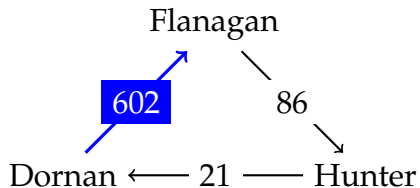




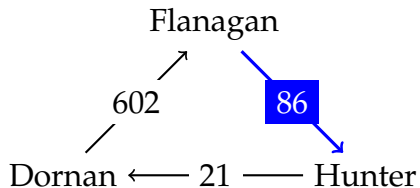




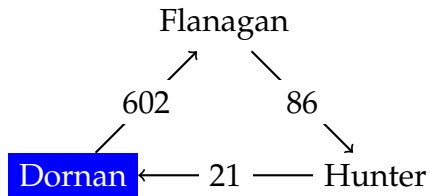
The largest head-to-head loss of Dornan is 21



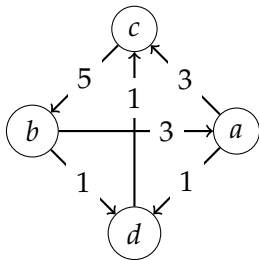
The largest head-to-head loss of Dornan is 21  
The largest head-to-head loss of Flanagan is 602

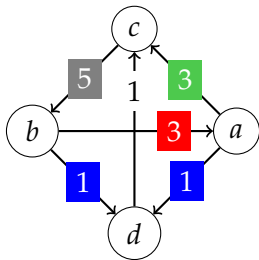


The largest head-to-head loss of Dornan is 21  
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The largest head-to-head loss of Hunter is 86

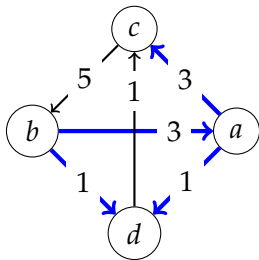


The largest head-to-head loss of Dornan is 21  
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The largest head-to-head loss of Hunter is 86  
Dornan is the Minimax winner.





*d* is the Minimax winner.



*d* is the Minimax winner.  
*a* and *b* are the Copeland winners.

# Split Cycle



1. In each majority cycle, identify the wins with the smallest margin in that cycle.



# Split Cycle

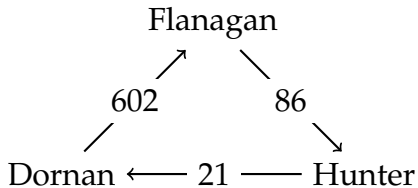


1. In each majority cycle, identify the wins with the smallest margin in that cycle.
2. After completing step 1 for all cycles, discard the identified wins. All remaining wins count as **defeats**.
3. The candidates that are not defeated by any other candidate are the Split Cycle winners.

# Split Cycle



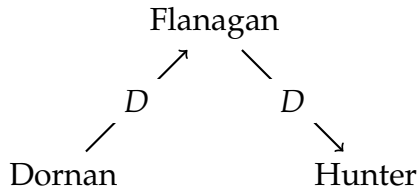
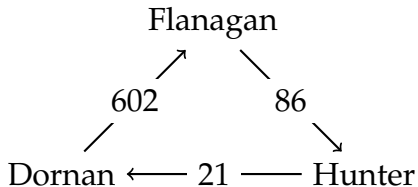
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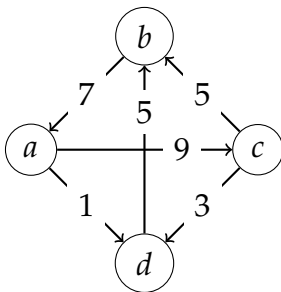


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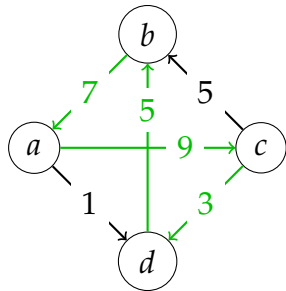
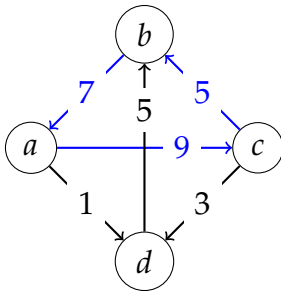
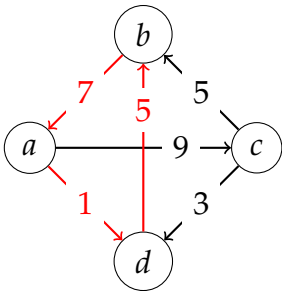
# Example

Suppose an election produces the following majority margin graph (i.e., there are 7 more voters who ranked  $b$  above  $a$  than who ranked  $a$  above  $b$ , etc.):

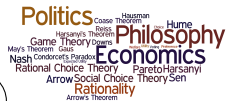
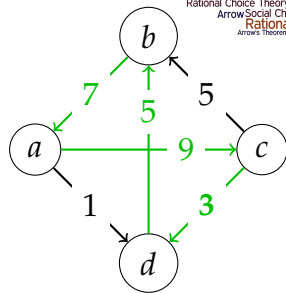
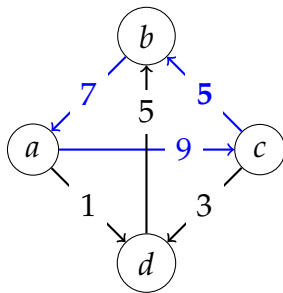
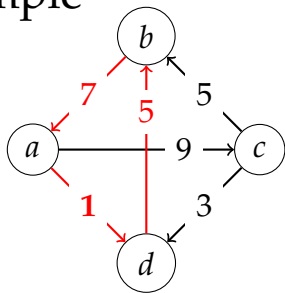




# Example

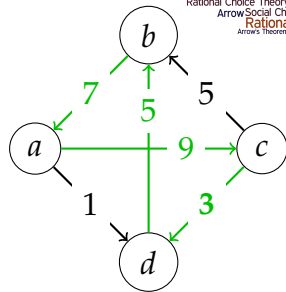
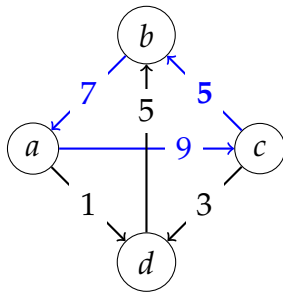
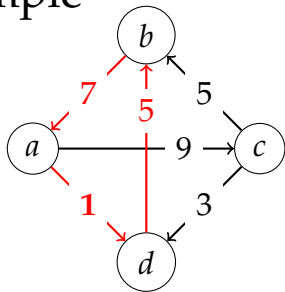


# Example



Next find the smallest margin in each cycle.

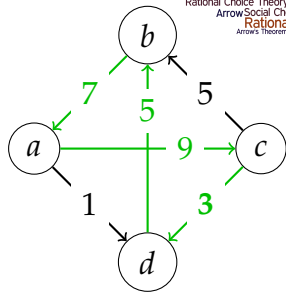
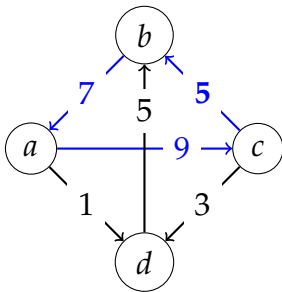
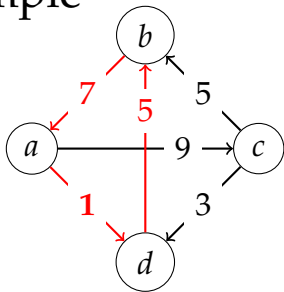
# Example



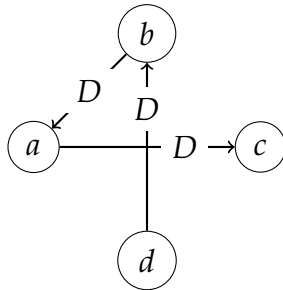
Next find the smallest margin in each cycle.  
These edges cannot be defeats.



# Example



Next find the smallest margin in each cycle.  
 These edges cannot be defeats.  
 But all other edges are defeats.  
 $d$  is the Split Cycle winner.



# Voting Method Tutorial



<https://voting-methods-8e393f57e966.herokuapp.com/>

# Which Voting Method is Best?



Voting Methods: Plurality, Borda, Instant Runoff Voting (Ranked Choice Voting), Coombs, Minimax, Copeland, Split Cycle

# Which Voting Method is Best?



Voting Methods: Plurality, Borda, Instant Runoff Voting (Ranked Choice Voting), Coombs, Minimax, Copeland, Split Cycle

- ▶ Voting methods that satisfy the top condition (winners must be ranked first by at least one voter): Plurality and Instant Runoff Voting
- ▶ Voting methods that always elect a Condorcet winner (when one exists): Minimax, Copeland, Split Cycle