PHPE 400
Individual and Group Decision Making

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Which Voting Method is Best?

- Voting methods that satisfy the top condition (winners must be ranked first by at least one voter): Plurality and Instant Runoff Voting

- Voting methods that always elect a Condorcet winner (when one exists): Minimax, Copeland, Maximum Wins-Smallest Loss

Majority Rule

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Majority Rule: a is ranked above (below) b if more (fewer) voters rank a above b than b above a, otherwise a and b are tied.

When there are only two options, can we argue that majority rule is the “best” procedure?

Two Principles of Group Decision Making

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2. **Equal Treatment**: Every voter should have the same opportunity to influence the result.

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Majority rule clearly satisfies both principles.

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Lottery Voting: Suppose there are two candidates, a and b. Each voter selects their preferred candidate (either a or b). Then, a single vote is randomly selected, and the candidate chosen on that ballot becomes the winner.

B. Saunders (2010). *Democracy, Political Equality, and Majority Rule*. Ethics, 121(1), pp. 148-177.

Justifying Majority Rule

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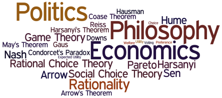
- Lottery Voting is not Majority Rule, since the vote of someone in the minority may be picked.

What Justifies Majority Rule?



M. Risse (2004). *Arguing for majority rule*. Journal of Political Philosophy 12 (1), pp. 41 - 64.

Justifying Majority Rule I

May's Theorem is a **procedualist justification** of majority rule showing that it is the unique procedure satisfying normative principles of group decision making.

K. May (1952). *A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision*. Econometrica, Vol. 20.

May's Theorem: Details

Voters: $V = \{1, 2, 3, \ldots, n\}$ is the set of n voters.

Candidates: $X = \{a, b\}$ is set of candidates.

Suppose that voters can submit one of 2 rankings:

1. $a\ P\ b$: a is ranked above b ("vote for a")
2. $b\ P\ a$: b is ranked above a ("vote for b")

Let $L(X)$ be the set of 2 rankings on X.

May's Theorem: Details

The set of **profiles** is $L(X)^V$, where a profile assigns to each voter one of the three rankings from $L(X)$.

Given a profile $\mathbf{P} \in L(X)^V$ and a voter $i \in V$, we write $\mathbf{P}_i$ for the ranking of voter i.

E.g., suppose that $V = \{1, 2, 3, 4\}$ and consider the profile

$$\mathbf{P} = (a\ P\ b, a\ P\ b, b\ P\ a, a\ P\ b)$$

Then, $\mathbf{P}_2$ is the ranking $a\ P\ b$ (voter 2 votes for a).

May's Theorem: Details

Social Choice Function: $F : L(X)^V \to \wp(X)$.

Where for all profiles $\mathbf{P}$ from $L(X)^V$, $F(\mathbf{P})$ is the set of winners.

We assume that for all profile $\mathbf{P}$, $F(\mathbf{P}) \neq \varnothing$ (so there is always at least one winner).

May's Theorem: Details

Social Choice Function: $F : L(X)^V \to \wp(X)$.

Examples:

- **Majority Rule**: The winner is the candidate with the most votes, otherwise the candidates are tied
- **Quota Rule**: The winner is the candidate with more than q% of the vote (e.g., more than 2/3 of the vote), otherwise the candidates are tied.
- **Unanimity Rule**: A candidate wins is *all voters* vote for that candidate, otherwise the candidates are tied.

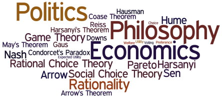
May's Theorem: Details

Social Choice Function: $F : L(X)^V \to \wp(X)$.

Examples:

- **Minority Rule**: The winner is the candidate with the fewest votes, otherwise the candidates are tied.
- **Majority Rule with Status Quo**: The winner is the candidate with the most votes, and if there is a tie candidate a wins.
- **Always a**: Candidate a always wins.
- **Voter 1**: The winner is whoever voter 1 voted for.
- **Tied**: The candidates are always tied.

May’s Theorem: Details

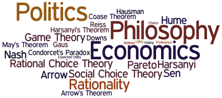


$$F_{Maj}(\mathbf{P}) = \begin{cases} \{a\} & \text{if more voters rank } a \text{ above } b \text{ than } b \text{ above } a \\ \{a, b\} & \text{if the same number of voters rank } a \text{ above } b \text{ as } b \text{ above } a \\ \{b\} & \text{if more voters rank } b \text{ above } a \text{ than } a \text{ above } b \end{cases}$$

May's Theorem: Details

$$F_{Maj}(\mathbf{P}) = \begin{cases} \{a\} & \text{if } Margin_{\mathbf{R}}(a,b) > 0 \\ \{a,b\} & \text{if } Margin_{\mathbf{R}}(a,b) = 0 \\ \{b\} & \text{if } Margin_{\mathbf{R}}(b,a) > 0 \end{cases}$$

Anonymity and Neutrality



- F satisfies **Anonymity**: permuting the voters does not change the set of winners.
- F satisfies **Neutrality**: permuting the candidates results in a winning set that is permuted in the same way.

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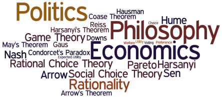

- F satisfies **Anonymity**: permuting the voters does not change the set of winners.
- F satisfies **Neutrality**: permuting the candidates results in a winning set that is permuted in the same way.

$\Longrightarrow$ in 2-candidate profiles, if the same number of voters rank $\boldsymbol{a}$ above $\boldsymbol{b}$ as $\boldsymbol{b}$ above $\boldsymbol{a}$, then $\boldsymbol{a} \in F(\mathbf{P})$ if, and only if, $\boldsymbol{b} \in F(\mathbf{P})$

($\boldsymbol{a}$ wins according to F if and only if $\boldsymbol{b}$ wins according to F).

Weak Positive Responsiveness

- F satisfies **Weak Positive Responsiveness** if for any profiles $\mathbf{P}$ and $\mathbf{P}'$, if

  1. $a \in F(\mathbf{P})$ (a is a winner in $\mathbf{P}$ according to F) and
  2. $\mathbf{P}'$ is obtained from $\mathbf{P}$ by one voter who ranked a uniquely last in $\mathbf{P}$ switching to ranking a uniquely first in $\mathbf{P}'$ (while all other voters' rankings are unchanged),

  then $F(\mathbf{P}') = \{a\}$ (a is the **unique** winner in $\mathbf{P}'$ according to F).