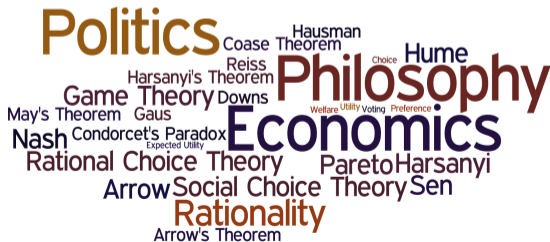


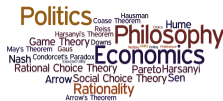
PHPE 400

Individual and Group Decision Making

Eric Pacuit
University of Maryland
pacuit.org



Iterative Voting Methods



Instant Runoff Voting (Ranked Choice): Iteratively remove all candidates with the **fewest number of voters who rank them first**, until there is a candidate with a majority of first-place votes. If, at some stage of the removal process, all remaining candidates have the same number of voters who rank them first (so all candidates would be removed), then all remaining candidates are selected as winners.

Coombs: Iteratively remove all candidates with the **most number of voters who rank them last**, until there is a candidate with a majority of first-place votes. If, at some stage of the removal process, all remaining candidates have the same number of voters who rank them last (so all candidates would be removed), then all remaining candidates are selected as winners.

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Instant Runoff winners

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Instant Runoff winners

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Instant Runoff winners d

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Instant Runoff winners *d*

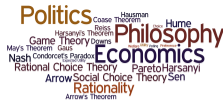
Coombs winners

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Instant Runoff winners *d*

Coombs winners *b*

2022 Alaska Special General Election



The Ranked Choice, also called Instant Runoff Voting (IRV), winner is Peltola.

- ▶ The write-ins are initially removed
- ▶ Begich is removed in the first round
- ▶ Palin loses to Peltola

2022 Alaska Special General Election



Round 1

Candidate	Votes	Percentage
Begich, Nick	53,810	28.53%
Palin, Sarah	58,973	31.27%
Peltola, Mary S.	75,799	40.19%
Continuing Ballots Total	188,582	
Blanks	3,412	
Exhausted	0	
Overvotes	295	
Remainder Points	0	
Non Transferable Total	3,707	

Begich, Nick is eliminated because the candidate had the least amount of votes.

2022 Alaska Special General Election



Elimination transfer for candidate Begich, Nick.

53810 ballots have been transferred in the following manner:

Transferred from	Transferred to	Ballots	Votes
Begich, Nick	Palin, Sarah	27053	27,053
Begich, Nick	Peltola, Mary S.	15467	15,467
Begich, Nick	Exhausted	11243	11,243
Begich, Nick	Overvotes	47	47

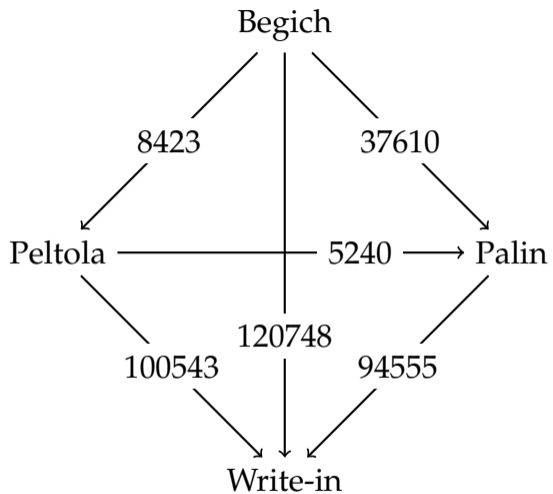
2022 Alaska Special General Election

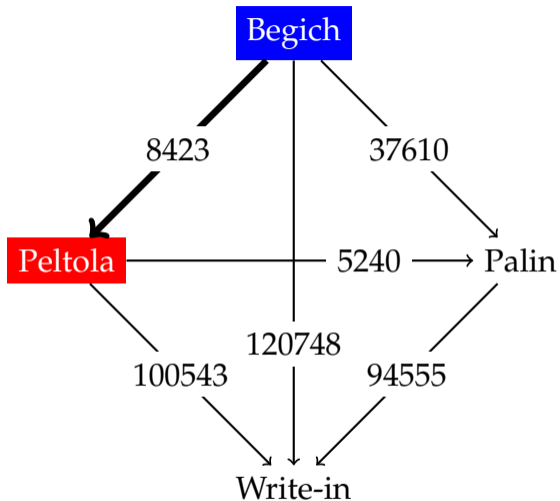


Round 2

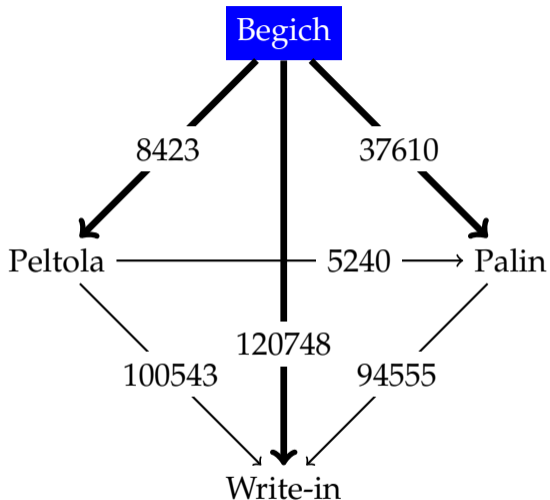
Candidate	Votes	Percentage
Begich, Nick	0	0.00%
Palin, Sarah	86,026	48.52%
Peltola, Mary S.	91,266	51.48%
Continuing Ballots Total	177,292	
Blanks	3,412	
Exhausted	11,243	
Overvotes	342	
Remainder Points	0	
Non Transferable Total	14,997	

Palin, Sarah is eliminated because the candidate was not elected in the last round.





- Problem 1: A majority of voters strictly prefer Begich to Peltola.



- ▶ Problem 1: A majority of voters strictly prefer Begich to Peltola.
- ▶ Problem 2: In fact, Begich is majority preferred to every other candidate, but is not elected.

Condorcet Winner



3	5	7	6
<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>d</i>	<i>b</i>
<i>c</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>d</i>	<i>d</i>	<i>a</i>	<i>a</i>

a

b

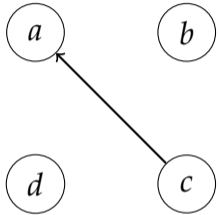
d

c

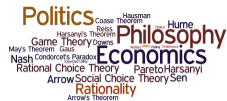
Condorcet Winner



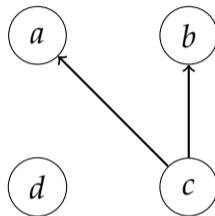
3	5	7	6
a	a	b	c
b	c	d	b
c	b	c	d
d	d	a	a



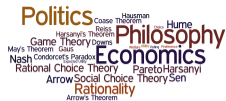
Condorcet Winner



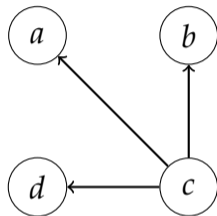
	3	5	7	6
<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>	
<i>b</i>	<i>c</i>	<i>d</i>	<i>b</i>	
<i>c</i>	<i>b</i>	<i>c</i>	<i>d</i>	
<i>d</i>	<i>d</i>	<i>a</i>	<i>a</i>	



Condorcet Winner



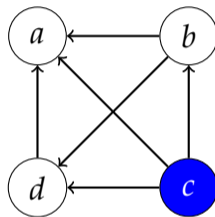
	3	5	7	6
<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>
<i>b</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>b</i>
<i>c</i>	<i>c</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>d</i>	<i>d</i>	<i>d</i>	<i>a</i>	<i>a</i>



Condorcet Winner



	3	5	7	6
<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>
<i>b</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>b</i>
<i>c</i>	<i>c</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>d</i>	<i>d</i>	<i>d</i>	<i>a</i>	<i>a</i>



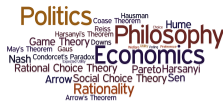
Condorcet Consistency



The **Condorcet winner** in a profile \mathbf{P} is a candidate x such that for all other candidates y , $\text{Margin}_{\mathbf{P}}(x, y) > 0$.

A voting method is **Condorcet consistent**, if for all \mathbf{P} , if x is a Condorcet winner in \mathbf{P} , then x is the unique winner according to the voting method.

Condorcet Consistency



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The **Condorcet loser** in a profile \mathbf{P} is a candidate x such that for all other candidates y , $\text{Margin}_{\mathbf{P}}(y, x) > 0$.

Some voting methods (e.g., Plurality) may elect a Condorcet loser; while other voting methods (e.g., Borda) are guaranteed to never elect a Condorcet loser.

Borda, Plurality, Plurality with Runoff, Instant Runoff Voting, Coombs are **not** Condorcet consistent.

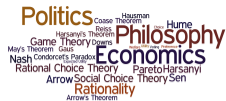
Can we find a voting method that is Condorcet consistent?

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Can we find a voting method that is Condorcet consistent?

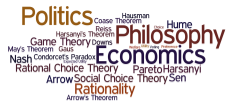
What about the method $F_{cond}(\mathbf{P}) = \{a\}$ where a is the Condorcet winner in \mathbf{P} ?

The Problem



Voter 1	Voter 2	Voter 3
<i>a</i>	<i>c</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>c</i>
<i>c</i>	<i>b</i>	<i>a</i>

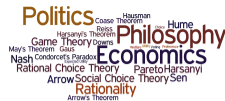
The Problem



Voter 1	Voter 2	Voter 3
<i>a</i>	<i>c</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>c</i>
<i>c</i>	<i>b</i>	<i>a</i>

- ▶ Does the group prefer *a* over *b*?

The Problem



Voter 1	Voter 2	Voter 3
<i>a</i>	<i>c</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>c</i>
<i>c</i>	<i>b</i>	<i>a</i>

- ▶ Does the group prefer *a* over *b*? **Yes**

The Problem



Voter 1	Voter 2	Voter 3
a	c	b
b	a	c
c	b	a

- ▶ Does the group prefer a over b ? **Yes**
- ▶ Does the group prefer b over c ? **Yes**

The Problem



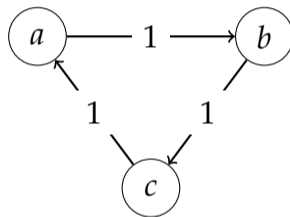
Voter 1	Voter 2	Voter 3
<i>a</i>	<i>c</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>c</i>
<i>c</i>	<i>b</i>	<i>a</i>

- ▶ Does the group prefer *a* over *b*? **Yes**
- ▶ Does the group prefer *b* over *c*? **Yes**
- ▶ Does the group prefer *a* over *c*? **No**

Majority Cycle Example



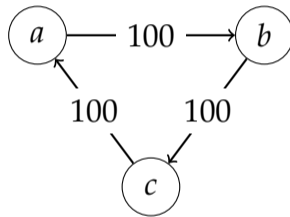
1	1	1
<i>a</i>	<i>c</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>c</i>
<i>c</i>	<i>b</i>	<i>a</i>



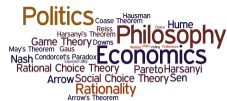
Majority Cycle Example



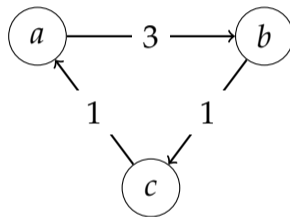
<u>100</u>	<u>100</u>	<u>100</u>
<i>a</i>	<i>c</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>c</i>
<i>c</i>	<i>b</i>	<i>a</i>



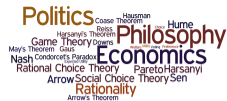
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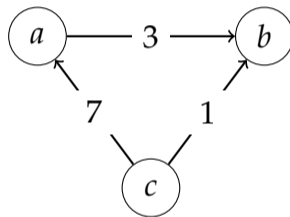
<u>2</u>	<u>2</u>	<u>1</u>
<i>a</i>	<i>c</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>c</i>
<i>c</i>	<i>b</i>	<i>a</i>



Not a Majority Cycle



1	5	3
<i>a</i>	<i>c</i>	<i>b</i>
<i>b</i>	<i>a</i>	<i>c</i>
<i>c</i>	<i>b</i>	<i>a</i>



Majority Cycles



A **majority cycle** is a list of candidates such that each has a positive margin over the next, and the last has a positive margin over the first.

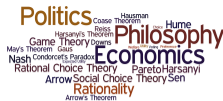
Majority Cycles



A **majority cycle** is a list of candidates such that each has a positive margin over the next, and the last has a positive margin over the first.

- ▶ Final decisions are extremely sensitive to institutional features such as who can set the agenda, arbitrary time limits place on deliberation, who is permitted to make motions, etc.

Majority Cycles



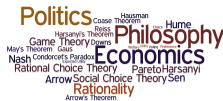
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- ▶ Is there *empirical evidence* that majority cycles have shown up in real elections?

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- ▶ How *likely* is a majority cycle?

Majority Cycles - Examples



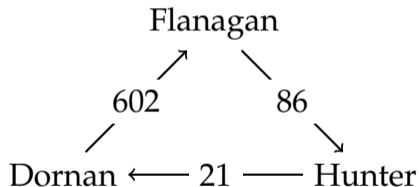
The 2007 Glasgow City Council election for Ward 5 (Govan): The election was run using Single-Transferable Vote to elect four candidates, but we can also imagine selecting a single winner based on these ballots.

Majority Cycles - Examples



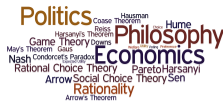
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The top three candidates were in a **majority cycle**:

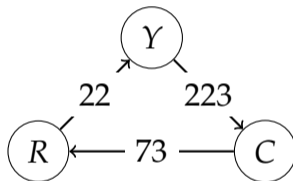


https://github.com/voting-tools/election-analysis/blob/main/glasgow_govan_2007.ipynb

Majority Cycles - Examples

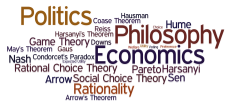


The 2021 Minneapolis City Council Election (Ward 2):



https://github.com/voting-tools/election-analysis/blob/main/minneapolis_2021.ipynb

Condorcet consistent voting methods



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Condorcet consistent voting methods

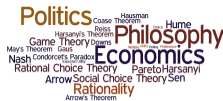


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We will study 3 Condorcet consistent voting methods: Copeland, Minimax, and Split Cycle.

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The Condorcet voting method Nanson was used in Marquette, Michigan, in the 1920s (Hoag and Hallett 1926, p. 491). To my knowledge, there are no cities using Condorcet consistent voting methods, but see the Condorcet Canada Initiative at <https://condorcet.ca>.