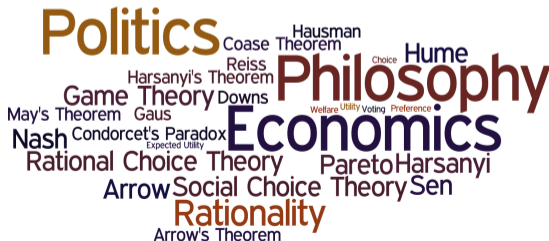# PHPE 400
Individual and Group Decision Making

Eric Pacuit
University of Maryland
pacuit.org

Voters | Rankings

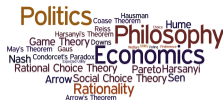1 — *a b c d*

2 — *b a d c*

3 — *b d a c*

4 — *d c a b*

Voting Method → Winning Set

There are many different voting methods

Plurality, Borda Count, Antiplurality/Veto; Coombs; (Strict/Weak) Nanson; Baldwin, Plurality with Runoff; Instant Runoff Voting; Copeland$_\alpha$; Bucklin; Minimax; Beat Path; Split Cycle; Stable Voting; Ranked Pairs; River; GETCHA; GOCHA; Kemeny; Dodgson Method; Young's Method; Approval Voting; Majority Judgment; Cumulative Voting; Range/Score Voting; . . .

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https://pref-voting.readthedocs.io/en/latest/collective_
decision_procedures.html
```

Electoral Reform

Politics Philosophy Economics word cloud (Coase Theorem, Hausman, Rolls, Hume, Harsanyi's Theorem, Downs, Game Theory, May's Theorem, Gauss, Condorcet's Paradox, Nash, Rational Choice Theory, Pareto, Harsanyi, Arrow, Social Choice Theory, Sen, Rationality, Arrow's Theorem)

POLITICO - PHOTOS · EDITOR
New York City Voters Just Adopted Ranked-Choice Voting in Elections. Here's How It Works

The Rules of the Game: A New Electoral System
Eric Maskin and Amartya Sen

POLITICO — MAGAZINE
ELECTIONS
New York's 'head-swirling' mistake puts harsh spotlight on ranked-choice voting
Two weeks without a winner in the tumultuous mayoral primary has ranked-choice backers desperate to maintain their momentum.

How to Depolarize American Politics
EDWARD B. FOLEY and ERIC S. MASKIN

Electoral Reform

- ► FairVote (http://www.fairvote.org)
- ► Center for Election Science (https://www.electology.org)
- ► Common Ground Democracy
  (https://edwardbfoley.substack.com/)

Electoral Reform

Politics Coase Hausman Philosophy Hume
Reiss Harsanyi's Theorem Gaus Game Theory Downs
May's Theorem Condorcet's Paradox Economics
Nash Rational Choice Theory Pareto Harsanyi
Arrow Social Choice Theory Sen
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- ▶ FairVote (`http://www.fairvote.org`)
- ▶ Center for Election Science (`https://www.electology.org`)
- ▶ Common Ground Democracy
  (`https://edwardbfoley.substack.com/`)
- ▶ Open primaries?
- ▶ Electoral college?
- ▶ How do you draw voting districts?

Electoral Reform: Proposition 83 in DC
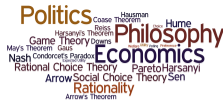
https://www.makeallvotescountdc.org/

Choosing how to choose

Pragmatic considerations: Is the procedure easy to use? Is it legal? The importance of *ease of use* should not be underestimated: Despite its many flaws, Plurality rule is, by far, the most commonly used method.

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Information required from the voters: What type of information do the ballots convey? I.e., Choosing a single alternative, linearly rank all the candidates, report something about the "intensity" of preference.

Choosing how to choose
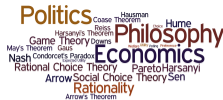
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Information required from the voters: What type of information do the ballots convey? I.e., Choosing a single alternative, linearly rank all the candidates, report something about the "intensity" of preference.

Axiomatics: Characterize the different voting methods in terms of normative principles of group decision making.

Notation
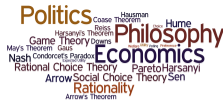
- *V* is a finite set of voters (assume that $V = \{1, 2, 3, \ldots, n\}$)

- *X* is a (typically finite) set of alternatives, or candidates

- An election **profile** is a record of the **ballot** submitted by each voter, where a ballot can be any of the following:
  - A selected candidate
  - A ranking of the candidates
  - Scores/grades assigned to each candidate

Rankings

MAYOR 市長	**1** 1st Choice 第一選擇	**2** 2nd Choice 第二選擇	**3** 3rd Choice 第三選擇	**4** 4th Choice 第四選擇	**5** 5th Choice 第五選擇	**6** 6th Choice 第六選擇
ELLEN LEE ZHOU / 李愛晨 Behavioral Health Clinician 行為健康臨床治療師	●	2	3	4	5	6
LONDON N. BREED / 倫敦·布里德 Mayor of San Francisco 三藩市市長	1	2	●	4	5	6
JOEL VENTRESCA / 喬爾·范崔斯卡 Retired Airport Analyst 退休機場分析師	1	2	3	4	●	6
WILMA PANG / 彭德慧 Retired Music Professor 退休音樂教授	1	2	3	●	5	6
ROBERT L. JORDAN, JR. / 小羅伯特·L·喬丹 Preacher 傳教士	1	2	3	4	5	●
PAUL YBARRA ROBERTSON / 保羅·伊巴拉·羅伯森 Small Business Owner 小企業業主	1	●	3	4	5	6
	1	2	3	4	5	6

Rankings

Politics Coase Theorem ... Hume
Harsanyi's Theorem Reiss Philosophy
Game Theory Downs
May's Theorem Gaus
Nash Condorcet's Paradox Economics
Rational Choice Theory Pareto Harsanyi
Arrow Social Choice Theory Sen
Rationality
Arrow's Theorem

Let X be a set of candidates and V a set of voters.

A **ranking** of X is a strict linear order P on X: a relation $P \subseteq X \times X$ satisfying the following conditions for all $x, y, z \in X$:

asymmetry: if $x\ P\ y$ then *not $y\ P\ x$*;

transitivity: if $x\ P\ y$ and $y\ P\ z$, then $x\ P\ z$;

weak completeness: if $x \neq y$, then $x\ P\ y$ or $y\ P\ x$.

Let $L(X)$ be the set of all strict linear orders on X.

Profiles

Politics Coase Theorem Hume Philosophy
Harsanyi's Theorem Rolls
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A **profile** for X is a function $\mathbf{P}$ assigning to $i \in V$ a linear order $\mathbf{P}_i$ on X.

Profiles

A **profile** for X is a function $\mathbf{P}$ assigning to $i \in V$ a linear order $\mathbf{P}_i$ on X.

So, $a\ \mathbf{P}_i\ b$ means that voter i ranks a above b, or that i strictly prefers candidate a to b.

For instance,

Example: let $V = \{v_1, v_2, v_3, v_4\}$ and $X = \{a, b, c, d\}$ and consider the following profile $\mathbf{P}$,

v_1	v_2	v_3	v_4
a	a	b	c
b	c	a	b
c	b	c	a

E.g., $a\ \mathbf{P}_{v_2}\ c$, $b\ \mathbf{P}_{v_4}\ a$, $a\ \mathbf{P}_{v_1}\ b$, ...

Anonymous Profiles
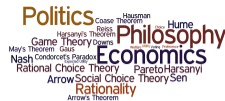
2	5	3	5
a	a	b	c
b	c	a	b
c	b	c	a

(Linear) Profiles

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}
b	b	b	b	b	b	b	a	a	a	a	a	a	a	a
c	c	c	c	c	c	c	c	c	c	c	c	b	b	b
a	a	a	a	a	a	a	b	b	b	b	b	c	c	c

(Linear) Anonymous Profile

Voting Method

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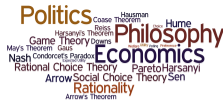
A **voting method** is a function that assigns a set of candidates (the winning set) to a profile.

Formally, a voting method is $F : L(X)^V \to \wp(X) \setminus \{\varnothing\}$, where $L(X)^V$ is the set of profiles of linear orders over X.

A voting method is **resolute** if for all profiles $\mathbf{P}$, $|F(\mathbf{P})| = 1$.

Majoritarianism
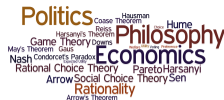
Politics Coase Theorem Hausman Hume Philosophy
Game Theory Rawls Downs
Harsanyi's Theorem
Nash Condorcet's Paradox Economics
May's Theorem Gaus
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Rationality
Arrow's Theorem

When there are only **two** candidates *a* and *b*, then all (reasonable) voting methods give the same results:

Majoritarianism
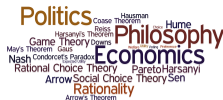
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Harsanyi's Theorem Hume
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When there are only **two** candidates a and b, then all (reasonable) voting methods give the same results:

Majority Rule: a is the winner if more than $1/2$ of the voters rank a above b, b is the winner if more than $1/2$ of votes rank b above a, otherwise a and b are tied.

Majoritarianism

Politics Coase Hausman
Theorem Rolls Hume
Harsanyi's Theorem Philosophy
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When there are only two options, can we argue that majority rule is the *best* procedure?

Majoritarianism

Politics Coase Theorem Rolls Hume Philosophy
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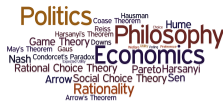Arrow Social Choice Theory Sen
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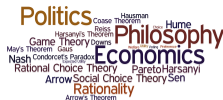Yes. We will look at two arguments: A procedural justification and an epistemic justification.

Majoritarianism

What about when there are *more than* two candidates, can we still argue that majority rule is the "best" procedure?

Majoritarianism

Politics Coase Theorem Hausman
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Majoritarianism

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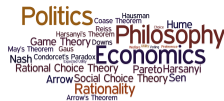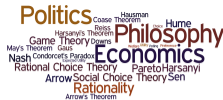Results are more mixed: Consider our previous definition of majority rule....we only defined it between two options! Can we generalize for $|X| > 2$?

The problem is that with more than 2 candidates, there may not be any candidate that is ranked first by more than half of the voters.

Positional scoring rules

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Harsanyi's Theorem Rawls
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A **scoring rule** each voter submits a ranking of the candidates. Based on the ranking, each voter assigns a *score* to each candidate. The candidates overall score is the sum of the scores assigned to the candidate by each voter. Then, the candidate(s) with the greatest overall score is(are) the winner(s).

Positional scoring rules
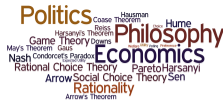
Politics Coase Theorem Rawls Hausman Hume Philosophy
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- ▶ **Plurality**: Each voter assigns a score of 1 to the candidate ranked in first place and 0 to all other candidates.

- ▶ **Borda**: If there are n candidates, then each voter assigns a score of $n - 1$ to the candidate in first place, $n - 2$ to the candidate in 2nd place, ..., and 0 to the candidate in last place.

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b

	7	5	4	3
1	a	b	d	c
0	b	c	b	d
0	c	d	c	a
0	d	a	a	b

Plurality winner(s): a

Plurality score of a: $1*7 + 0*0 + 0*3 + 0*9 = 7$
Plurality score of b: $1*5 + 0*11 + 0*0 + 0*3 = 5$
Plurality score of c: $1*4 + 0*5 + 0*11 + 0*0 = 4$
Plurality score of d: $1*3 + 0*3 + 0*5 + 0*7 = 3$

	7	5	4	3
3	a	b	d	c
2	b	c	b	d
1	c	d	c	a
0	d	a	a	b

Borda winner(s): b

Borda score of a: $\quad 3*7 \quad + \quad 2*0 \quad + \quad 1*3 \quad + \quad 0*9 \quad = \quad 24$
Borda score of b: $\quad 3*5 \quad + \quad 2*11 \quad + \quad 1*0 \quad + \quad 0*3 \quad = \quad 37$
Borda score of c: $\quad 3*4 \quad + \quad 2*5 \quad + \quad 1*11 \quad + \quad 0*0 \quad = \quad 33$
Borda score of d: $\quad 3*3 \quad + \quad 2*3 \quad + \quad 1*5 \quad + \quad 0*7 \quad = \quad 20$

1	2	2
x	y	y
y	x	x

Who are the Borda winners? y

1	2	2
x	y	y
a_1	x	x
a_2	a_1	a_1
a_3	a_2	a_2
y	a_3	a_3

Who are the Borda winners?

1	2	2
x	y	y
a_1	x	x
a_2	a_1	a_1
a_3	a_2	a_2
y	a_3	a_3

Who are the Borda winners? x and y

1	2	2
x	y	y
a_1	x	x
a_2	a_1	a_1
a_3	a_2	a_2
a_4	a_3	a_3
y	a_4	a_4

Who are the Borda winners?

1	2	2
x	y	y
a_1	x	x
a_2	a_1	a_1
a_3	a_2	a_2
a_4	a_3	a_3
y	a_4	a_4

Who are the Borda winners? x,
**but a majority of voters prefer y over x.**

When there is no majority winner, can we find the candidate(s) that is(are) "closest" to the majority winner?

Let's start with an example involving the voting method known as "**Ranked Choice Voting**," "**Instant Runoff**," or "**Hare System**."

This is widely used in Australia and is promoted in the U.S. by FairVote.org and the anti-corruption campaign RepresentUs.

Instant Runoff (aka Ranked Choice)
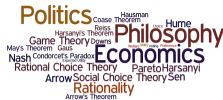
Iteratively remove all candidates with the fewest number of voters who rank them first, until there is a candidate with a majority of first-place votes. If, at some stage of the removal process, all remaining candidates have the same number of voters who rank them first (so all candidates would be removed), then all remaining candidates are selected as winners.

Coombs

Iteratively remove all candidates with the most number of voters who rank them last, until there is a candidate with a majority of first-place votes. If, at some stage of the removal process, all remaining candidates have the same number of voters who rank them last (so all candidates would be removed), then all remaining candidates are selected as winners.

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b

Instant Runoff winners

7	5	4	3
a	*b*	*d*	*c*
b	*c*	*b*	*d*
c	*d*	*c*	*a*
d	*a*	*a*	*b*

Instant Runoff winners

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b

Instant Runoff winners   *d*

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b

Instant Runoff winners d

Coombs winners

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b

Instant Runoff winners d

Coombs winners b