PHPE 400
Individual and Group Decision Making

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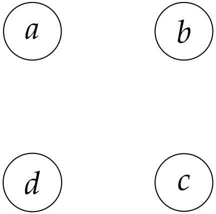
University of Maryland
pacuit.org

Try it: Vote by ranking

https://consensus-choice.pacuit.org/vote/ice-cream-poll

Example

3	5	7	6
a	a	b	c
b	c	d	b
c	b	c	d
d	d	a	a



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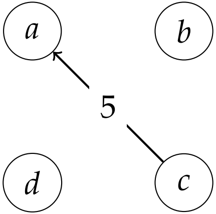

$$Margin_{\mathbf{P}}(a, c) = 8 - 13 = -5$$

$$Margin_{\mathbf{P}}(c, a) = 13 - 8 = 5$$

Example

3	5	7	6
a	a	b	c
b	c	d	b
c	b	c	d
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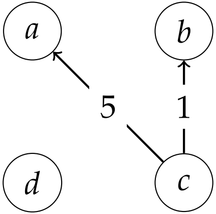
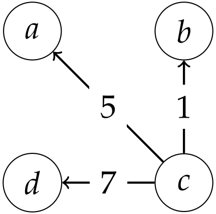


$$Margin_{\mathbf{P}}(b, c) = 10 - 11 = -1$$

$$Margin_{\mathbf{P}}(c, b) = 11 - 10 = 1$$

Example

3	5	7	6
a	a	b	c
b	c	d	b
c	b	c	d
d	d	a	a

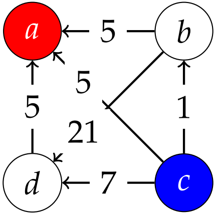


$$Margin_{\mathbf{P}}(c, d) = 14 - 7 = 7$$

$$Margin_{\mathbf{P}}(d, c) = 7 - 14 = -7$$

Example

3	5	7	6
a	a	b	c
b	c	d	b
c	b	c	d
d	d	a	a



c is the Condorcet winner

a is the Condorcet loser

The Problem

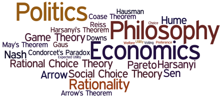
Voter 1	Voter 2	Voter 3
a	c	b
b	a	c
c	b	a

The Problem

Voter 1	Voter 2	Voter 3
a	c	b
b	a	c
c	b	a

- Does the group prefer a over b?

The Problem



Voter 1	Voter 2	Voter 3
a	*c*	*b*
b	*a*	*c*
c	*b*	*a*

- Does the group prefer *a* over *b*? Yes

The Problem

Voter 1	Voter 2	Voter 3
a	c	b
b	a	c
c	b	a

- Does the group prefer a over b? Yes
- Does the group prefer b over c? Yes

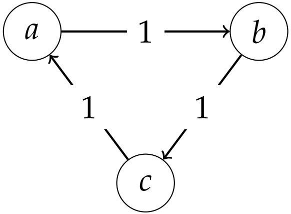
The Problem

Voter 1	Voter 2	Voter 3
a	c	b
b	a	c
c	b	a

- Does the group prefer a over b? Yes
- Does the group prefer b over c? Yes
- Does the group prefer a over c? No

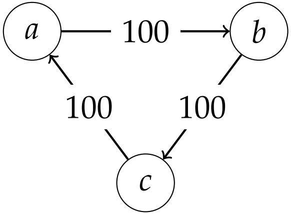
Majority Cycle Example

1	1	1
a	c	b
b	a	c
c	b	a



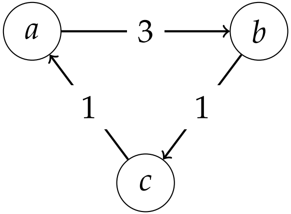
Majority Cycle Example

100	100	100
a	c	b
b	a	c
c	b	a



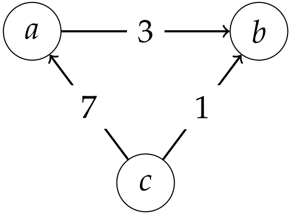
Majority Cycle Example

2	2	1
a	c	b
b	a	c
c	b	a



Not a Majority Cycle

1	5	3
a	c	b
b	a	c
c	b	a



Majority Cycles

A **majority cycle** is a list of candidates such that each has a positive margin over the next, and the last has a positive margin over the first.

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- Is there *empirical evidence* that majority cycles have shown up in real elections?

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- How *likely* is a majority cycle?

Majority Cycles - Examples

The 2007 Glasgow City Council election for Ward 5 (Govan): The election was run using Single-Transferable Vote to elect four candidates, but we can also imagine selecting a single winner based on these ballots.

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The top three candidates were in a **majority cycle**:

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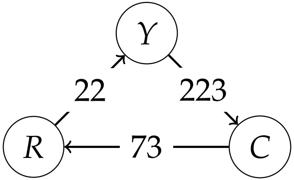
\begin{tikzcd}
 & \text{Flanagan} \arrow[dr, "86"] & \\
\text{Dornan} \arrow[ur, "602"] & & \text{Hunter} \arrow[ll, "21"]
\end{tikzcd}
$$

`https://github.com/voting-tools/election-analysis/blob/main/glasgow_govan_2007.ipynb`

Majority Cycles - Examples

The 2021 Minneapolis City Council Election (Ward 2):



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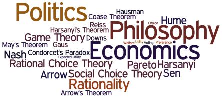
Condorcet consistent voting methods



The **Condorcet winner** in a profile $\mathbf{P}$ is a candidate x such that for all other candidates y, $Margin_{\mathbf{P}}(x, y) > 0$.

A voting method is **Condorcet consistent**, if for all $\mathbf{P}$, if x is a Condorcet winner in $\mathbf{P}$, then x is the unique winner according to the voting method.

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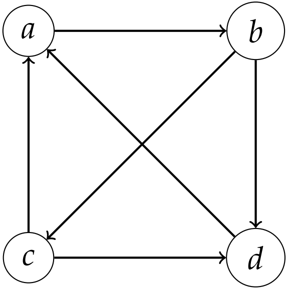
We will study 3 Condorcet consistent voting methods: Copeland, Minimax, and Maximum Win-Smallest Loss.

Copeland

Say that the **win-loss record** for a candidate x is the number of candidates that x is majority preferred to minus the number of candidates that is majority preferred to y.

Then, any candidate with the largest win-loss record is a Copeland winner.

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b



7	5	4	3
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b	c	b	d
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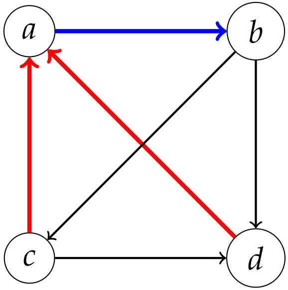
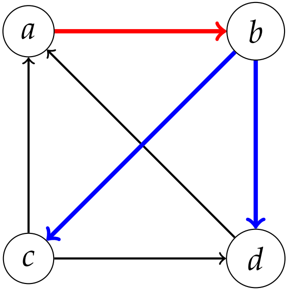


Win-loss record for a: $1 - 2 = -1$

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b



Win-loss record for a: $1 - 2 = -1$

Win-loss record for b: $2 - 1 = 1$

7	5	4	3
a	b	d	c
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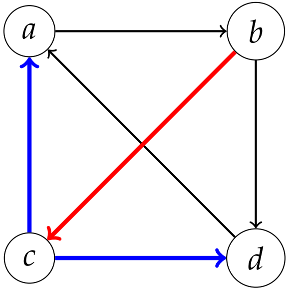

Win-loss record for a: $1 - 2 = -1$

Win-loss record for b: $2 - 1 = 1$

Win-loss record for c: $2 - 1 = 1$

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
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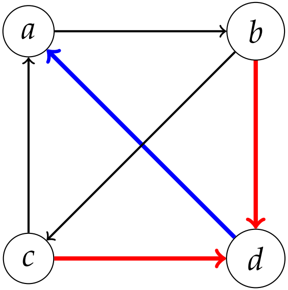


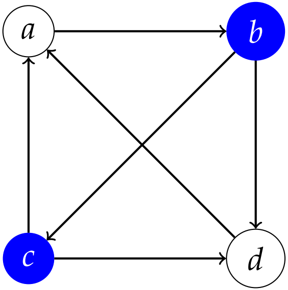
Win-loss record for a: $1 - 2 = -1$

Win-loss record for b: $2 - 1 = 1$

Win-loss record for c: $2 - 1 = 1$

Win-loss record for d: $1 - 2 = -1$

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b



Win-loss record for a: $1 - 2 = -1$
Win-loss record for b: $2 - 1 = 1$
Win-loss record for c: $2 - 1 = 1$
Win-loss record for d: $1 - 2 = -1$
c and b are the Copeland winners.

2007 Glasgow City Council

The top three candidates were in a **majority cycle**:

$$\begin{tikzcd}
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All candidates are tied according to Copeland (each candidate's win-loss record is 0).

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It's Dornan.

Minimax

Say that the head-to-head loss of x vs. y is the margin of y over x: the number of voters that rank y above x minus the number of voters that rank x above y.

Find the largest head-to-head loss for each candidate. Any candidate with the smallest such loss is a Minimax winner.

$$
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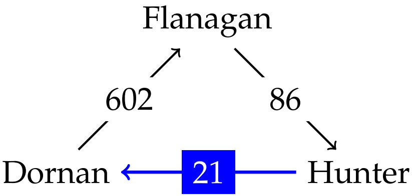
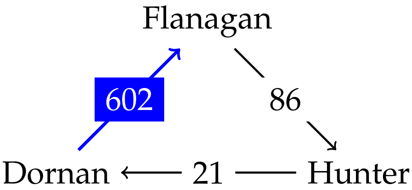


The largest head-to-head loss of Dornan is 21



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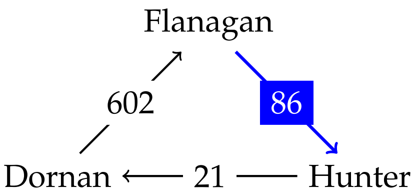

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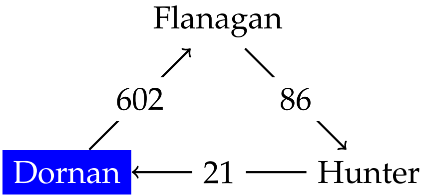
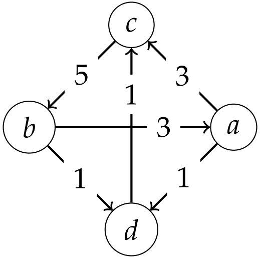
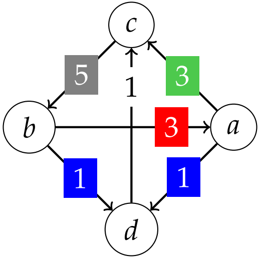
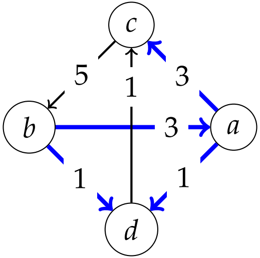


The largest head-to-head loss of Dornan is 21
The largest head-to-head loss of Flanagan is 602
The largest head-to-head loss of Hunter is 86
Dornan is the Minimax winner.


c
5
1
3
b
3
a
1
1
d



d is the Minimax winner.



d is the Minimax winner.
a and b are the Copeland winners.

Most Wins, Smallest Loss

The winner is the candidate with the most head-to-head wins.
If multiple candidates tie for the most wins, return the one with the smallest head-to-head loss.

7	3	1	4	5	1
a	b	c	c	d	d
c	a	b	b	b	c
d	d	a	d	a	b
b	c	d	a	c	a

W. Holliday (2025). *A Simple Condorcet Voting Method for Final Four Elections*. Representation, 61(4), pp. 483–506. `https://doi.org/10.1080/00344893.2025.2473397`.

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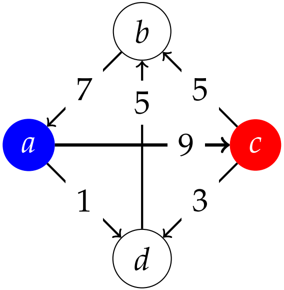
c	a	b	b	b	c
d	d	a	d	a	b
b	c	d	a	c	a



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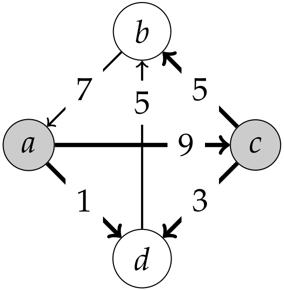
7	3	1	4	5	1
a	*b*	*c*	*c*	*d*	*d*
c	*a*	*b*	*b*	*b*	*c*
d	*d*	*a*	*d*	*a*	*b*
b	*c*	*d*	*a*	*c*	*a*



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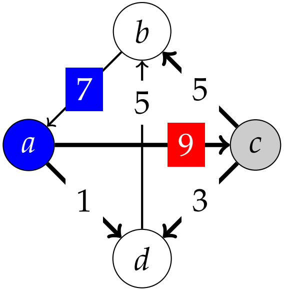
d	d	a	d	a	b
b	c	d	a	c	a



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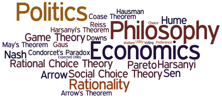
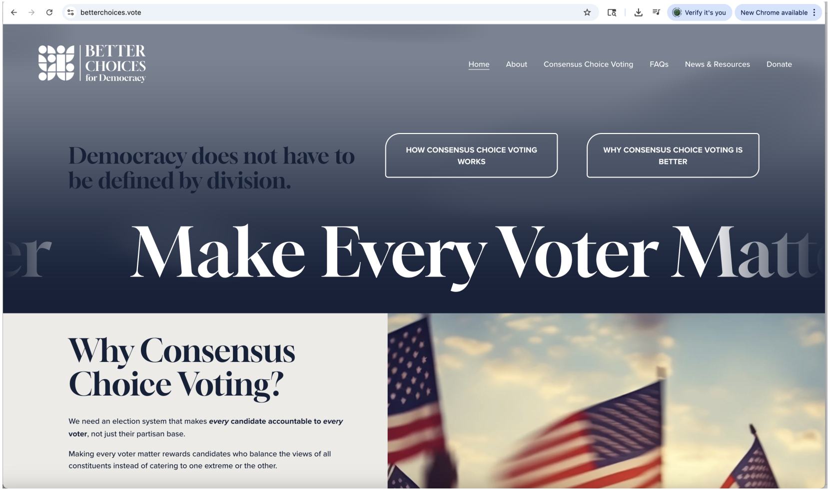


betterchoices.vote

Elect the **Condorcet Winner**: the candidate who wins every head-to-head comparison.



Electoral Reform

- FairVote
  http://www.fairvote.org
- Center for Election Science
  https://www.electology.org
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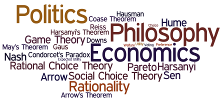
- Open primaries?
- Electoral college?
- How do you draw voting districts?
- Proportional representation?

Which Voting Method is Best?

- Voting methods that satisfy the top condition (winners must be ranked first by at least one voter): Plurality and Instant Runoff Voting

- Voting methods that always elect a Condorcet winner (when one exists): Minimax, Copeland, Maximum Wins-Smallest Loss

PHPE 4080/PHIL 438V: The Theory of Voting



PHPE408O **Advanced Topics in Philosophy, Politics, and Economics; The Theory of Voting**

Syllabus Repository (0)

Credits: 3 Grading Method: Regular, Pass-Fail

Cross-listed with PHIL438V. Credit granted only for PHPE408O or PHIL438V.

When friends disagree about where to go for dinner or citizens choose between political candidates, how should the group decide? This course investigates the theory behind voting and collective decision-making, examining different voting methods, surprising paradoxes that arise in elections, and fundamental results about what fair group decisions can and cannot achieve. No prior background in social choice theory is assumed, though familiarity with rational choice theory and/or game theory will be helpful (e.g., PHPE 400, GVPT 390, or introductory microeconomics).

Show Sections

Majority Rule

When there are only **two** candidates a and b, then all voting methods give the same results

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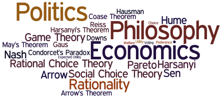
Majority Rule: a is ranked above (below) b if more (fewer) voters rank a above b than b above a, otherwise a and b are tied.

When there are only two options, can we argue that majority rule is the "best" procedure?

Two Principles of Group Decision Making



1. **Democratic Responsiveness**: The outcome should reflect and respond to voters' preferences.
2. **Equal Treatment**: Every voter should have the same opportunity to influence the result.

Two Principles of Group Decision Making

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2. **Equal Treatment**: Every voter should have the same opportunity to influence the result.

Majority rule clearly satisfies both principles.

Two Principles of Group Decision Making

However, Majority Rule is not the only procedure that satisfies these principles.

Lottery Voting: Suppose there are two candidates, a and b. Lottery Voting proceeds as follows: Each voter selects their preferred candidate (either a or b). A single vote is then randomly selected, and the candidate chosen on that ballot becomes the winner.

B. Saunders (2010). *Democracy, Political Equality, and Majority Rule*. Ethics, 121(1), pp. 148-177.

Justifying Majority Rule

- Lottery Voting satisfies Democratic Responsiveness, since all members of the community have a chance to influence outcomes

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- Lottery Voting satisfies Equal Treatment, since all voters have an equal chance of being picked.

  It gives each voter an equal chance of being decisive, but voters do not have equal chances of getting their way—rather, the chance of each option winning is proportional to the number of votes it obtains.

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- Lottery Voting satisfies Democratic Responsiveness, since all members of the community have a chance to influence outcomes

- Lottery Voting satisfies Equal Treatment, since all voters have an equal chance of being picked.

  It gives each voter an equal chance of being decisive, but voters do not have equal chances of getting their way—rather, the chance of each option winning is proportional to the number of votes it obtains.

- Lottery Voting is not Majority Rule, since the vote of someone in the minority may be picked.

What Justifies Majority Rule?

M. Risse (2004). *Arguing for majority rule*. Journal of Political Philosophy 12 (1), pp. 41 - 64.

Justifying Majority Rule

When there are only two options, can we argue that majority rule is the "best" procedure?

Setting aside the possibility of using lotteries, May's Theorem is a proceduralist justification of majority rule showing that it is the unique procedure satisfying normative principles of group decision making.

K. May (1952). *A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision*. Econometrica, Vol. 20.

May's Theorem: Details

Voters: $V = \{1, 2, 3, \ldots, n\}$ is the set of n voters.

Candidates: $X = \{a, b\}$ is set of candidates.

Suppose that voters can submit one of 2 rankings:

1. $a \; P \; b$: a is ranked above b ("vote for a")
2. $b \; P \; a$: b is ranked above a ("vote for b")

Let $L(X)$ be the set of 2 rankings on X.

May's Theorem: Details

The set of **profiles** is $L(X)^V$, where a profile assigns to each voter one of the three rankings from $L(X)$.

Given a profile $\mathbf{P} \in L(X)^V$ and a voter $i \in V$, we write $\mathbf{P}_i$ for the ranking of voter i.

E.g., suppose that $V = \{1, 2, 3, 4\}$ and consider the profile

$$\mathbf{P} = (a\ P\ b, a\ P\ b, b\ P\ a, a\ P\ b)$$

Then, $\mathbf{P}_2$ is the ranking $a\ P\ b$ (voter 2 votes for a).

May's Theorem: Details

Social Choice Function: $F : L(X)^V \to \wp(X)$.

Where for all profiles $\mathbf{P}$ from $L(X)^V$, $F(\mathbf{P})$ is the set of winners.

We assume that for all profile $\mathbf{P}$, $F(\mathbf{P}) \neq \varnothing$ (so there is always at least one winner).

May's Theorem: Details

Social Choice Function: $F : L(X)^V \to \wp(X)$.

Examples:

- **Majority Rule**: The winner is the candidate with the most votes, otherwise the candidates are tied
- **Quota Rule**: The winner is the candidate with more than q% of the vote (e.g., more than 2/3 of the vote), otherwise the candidates are tied.
- **Unanimity Rule**: A candidate wins is *all voters* vote for that candidate, otherwise the candidates are tied.

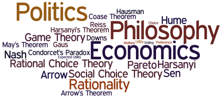
May’s Theorem: Details

Social Choice Function: $F : L(X)^V \rightarrow \wp(X)$.

Examples:

- **Minority Rule**: The winner is the candidate with the fewest votes, otherwise the candidates are tied.
- **Majority Rule with Status Quo**: The winner is the candidate with the most votes, and if there is a tie candidate a wins.
- **Always a**: Candidate a always wins.
- **Voter 1**: The winner is whoever voter 1 voted for.
- **Tied**: The candidates are always tied.

May’s Theorem: Details



$$F_{Maj}(\mathbf{P}) = \begin{cases} \{a\} & \text{if more voters rank } a \text{ above } b \text{ than } b \text{ above } a \\ \{a, b\} & \text{if the same number of voters rank } a \text{ above } b \text{ as } b \text{ above } a \\ \{b\} & \text{if more voters rank } b \text{ above } a \text{ than } a \text{ above } b \end{cases}$$

May's Theorem: Details

$$F_{Maj}(\mathbf{P}) = \begin{cases} \{a\} & \text{if } Margin_{\mathbf{R}}(a,b) > 0 \\ \{a,b\} & \text{if } Margin_{\mathbf{R}}(a,b) = 0 \\ \{b\} & \text{if } Margin_{\mathbf{R}}(b,a) > 0 \end{cases}$$

Anonymity and Neutrality

- F satisfies **Anonymity**: permuting the voters does not change the set of winners.
- F satisfies **Neutrality**: permuting the candidates results in a winning set that is permuted in the same way.

Anonymity and Neutrality

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$\Longrightarrow$ in 2-candidate profiles, if the same number of voters rank $\boldsymbol{a}$ above $\boldsymbol{b}$ as $\boldsymbol{b}$ above $\boldsymbol{a}$, then $\boldsymbol{a} \in F(\mathbf{P})$ if, and only if, $\boldsymbol{b} \in F(\mathbf{P})$

($\boldsymbol{a}$ wins according to F if and only if $\boldsymbol{b}$ wins according to F).

Weak Positive Responsiveness

- F satisfies **Weak Positive Responsiveness** if for any profiles $\mathbf{P}$ and $\mathbf{P}'$, if
  1. $a \in F(\mathbf{P})$ (a is a winner in $\mathbf{P}$ according to F) and
  2. $\mathbf{P}'$ is obtained from $\mathbf{P}$ by one voter who ranked a uniquely last in $\mathbf{P}$ switching to ranking a uniquely first in $\mathbf{P}'$ (while all other voters' rankings are unchanged),

  then $F(\mathbf{P}') = \{a\}$ (a is the **unique** winner in $\mathbf{P}'$ according to F).

Below is all possible profiles for 3 voters and two candidates a and b, and the outcomes of different voting methods.

Profile	Voter 1	Always a	Minority	Unanimity	Majority
$(a\ P\ b, a\ P\ b, a\ P\ b)$	a	a	b	a	a
$(a\ P\ b, a\ P\ b, b\ P\ a)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a, a\ P\ b)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a, b\ P\ a)$	a	a	a	a, b	b
$(b\ P\ a, a\ P\ b, a\ P\ b)$	b	a	b	a, b	a
$(b\ P\ a, a\ P\ b, b\ P\ a)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a, a\ P\ b)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a, b\ P\ a)$	b	a	a	b	b

Profile	Voter 1	Always a	Minority	Unanimity	Majority
$(a\ P\ b, a\ P\ b, a\ P\ b)$	a	a	b	a	a
$(a\ P\ b, a\ P\ b, b\ P\ a)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a, a\ P\ b)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a, b\ P\ a)$	a	a	a	a, b	b
$(b\ P\ a, a\ P\ b, a\ P\ b)$	b	a	b	a, b	a
$(b\ P\ a, a\ P\ b, b\ P\ a)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a, a\ P\ b)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a, b\ P\ a)$	b	a	a	b	b

Voter 1 violates Anonymity: The method **Voter 1** assigns different winners to the profiles $(a\ P\ b, a\ P\ b, b\ P\ a)$ and $(b\ P\ a, a\ P\ b, a\ P\ b)$.

Profile	Voter 1	Always a	Minority	Unanimity	Majority
$(a\ P\ b, a\ P\ b, a\ P\ b)$	a	a	b	a	a
$(a\ P\ b, a\ P\ b, b\ P\ a)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a, a\ P\ b)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a, b\ P\ a)$	a	a	a	a, b	b
$(b\ P\ a, a\ P\ b, a\ P\ b)$	b	a	b	a, b	a
$(b\ P\ a, a\ P\ b, b\ P\ a)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a, a\ P\ b)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a, b\ P\ a)$	b	a	a	b	b

Always a violates Neutrality: The method **Always** a assigns a as a winner to $(a\ P\ b, a\ P\ b, b\ P\ a)$, and assigns a to $(b\ P\ a, b\ P\ a, a\ P\ b)$ (but it should assign b according to Neutrality).

Profile	Voter 1	Always a	Minority	Unanimity	Majority
$(a\ P\ b, a\ P\ b, a\ P\ b)$	a	a	b	a	a
$(a\ P\ b, a\ P\ b, b\ P\ a)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a, a\ P\ b)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a, b\ P\ a)$	a	a	a	a, b	b
$(b\ P\ a, a\ P\ b, a\ P\ b)$	b	a	b	a, b	a
$(b\ P\ a, a\ P\ b, b\ P\ a)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a, a\ P\ b)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a, b\ P\ a)$	b	a	a	b	b

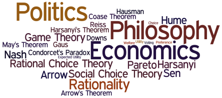
Minority violates Weak Positive Responsiveness: b is a winner in $(b\ P\ a, a\ P\ b, a\ P\ b)$, but $(b\ P\ a, b\ P\ a, a\ P\ b)$ is a profile in which one voter (voter 2) moves b from the bottom to the top of their ranking yet b does not win in this profile.

Profile	Voter 1	Always a	Minority	Unanimity	Majority
$(a\ P\ b, a\ P\ b, a\ P\ b)$	a	a	b	a	a
$(a\ P\ b, a\ P\ b, b\ P\ a)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a, a\ P\ b)$	a	a	b	a, b	a
$(a\ P\ b, b\ P\ a, b\ P\ a)$	a	a	a	a, b	b
$(b\ P\ a, a\ P\ b, a\ P\ b)$	b	a	b	a, b	a
$(b\ P\ a, a\ P\ b, b\ P\ a)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a, a\ P\ b)$	b	a	a	a, b	b
$(b\ P\ a, b\ P\ a, b\ P\ a)$	b	a	a	b	b

Unanimity violates Weak Positive Responsiveness: b is a winner in $(b\ P\ a, a\ P\ b, a\ P\ b)$, but $(b\ P\ a, b\ P\ a, a\ P\ b)$ is a profile in which one voter (voter 2) moves b from the bottom to the top of their ranking yet b **is not the unique winner** in this profile.

	Anonymity	Neutrality	Weak Positive Responsiveness
Voter 1	✗	✓	✗
Always *a*	✓	✗	✓
Minority Rule	✓	✓	✗
Unanimity Rule	✓	✓	✗
Majority Rule	✓	✓	✓

May's Theorem



Theorem (May 1952)

Let F be a voting method on the domain of two-alternative profiles. Then the following are equivalent:

1. F satisfies Anonymity, Neutrality, and Weak Positive Responsiveness;
2. F is Majority Rule.

Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness.
Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$?

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Suppose that $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$

By Neutrality, $F(b\ P\ a, b\ P\ a, a\ P\ b) = \{a\}$

Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness. Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$? No!

Suppose that $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$

By Neutrality, $F(b\ P\ a, b\ P\ a, a\ P\ b) = \{a\}$

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Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness.
Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$? No!

Suppose that $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$

By Neutrality, $F(b\ P\ a, b\ P\ a, a\ P\ b) = \{a\}$

By Anonymity, $F(a\ P\ b, b\ P\ a, b\ P\ a) = \{a\}$

By Weak Positive Responsiveness, $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a\}$

Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness. Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$? No!

Suppose that $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$

By Neutrality, $F(b\ P\ a, b\ P\ a, a\ P\ b) = \{a\}$

By Anonymity, $F(a\ P\ b, b\ P\ a, b\ P\ a) = \{a\}$

By Weak Positive Responsiveness, $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a\}$

Contradiction: Since F is a function, we can't have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{b\}$ and $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a\}$

Proof Sketch

Suppose that F satisfies Anonymity, Neutrality and Positive Responsiveness. Can we have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a, b\}$? No!

Suppose that $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a, b\}$

By Neutrality, $F(b\ P\ a, b\ P\ a, a\ P\ b) = \{a, b\}$

By Anonymity, $F(a\ P\ b, b\ P\ a, b\ P\ a) = \{a, b\}$

By Weak Positive Responsiveness, $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a\}$

Contradiction: Since F is a function, we can't have $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a, b\}$ and $F(a\ P\ b, a\ P\ b, b\ P\ a) = \{a\}$

May's Theorem is a *proceduralist* justification of majority rule showing that Majority Rule is the unique group decision method satisfying two basic principles of fairness (Anonymity and Neutrality) and a basic principle ensuring that the outcome responds appropriately to the voters' opinions (Weak Positive Responsiveness).

May’s Theorem is a *proceduralist* justification of majority rule showing that Majority Rule is the unique group decision method satisfying two basic principles of fairness (Anonymity and Neutrality) and a basic principle ensuring that the outcome responds appropriately to the voters’ opinions (Weak Positive Responsiveness).

We can also give an *epistemic* justification of majority rule showing that has a high probability of identifying the correct answer to a question.