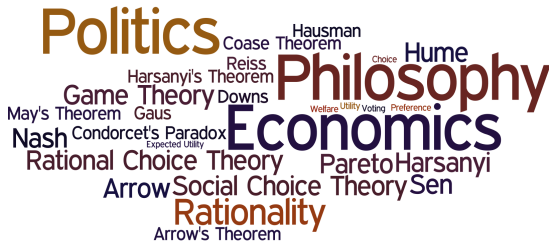


# PHPE 400

## Individual and Group Decision Making

Eric Pacuit  
University of Maryland  
[pacuit.org](http://pacuit.org)

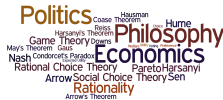


# Voting Methods

## Positional Scoring Rules: Plurality, Borda, ...

**Iterative Voting Methods:** Iteratively remove “poorly performing candidates” until there is a candidate with a majority of first-place votes.

# Instant Runoff (aka Ranked Choice)



Iteratively remove all candidates with **the fewest number of voters who rank them first**, until there is a candidate with a majority of first-place votes.

If, at some stage of the removal process, all remaining candidates have the same number of voters who rank them first (so all candidates would be removed), then all remaining candidates are selected as winners.

← → 🔍 fairvote.org ☆ 📄 📌 🗨️ 🔒 Verify it's you New Chrome available

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## PROTECTING THE VOTING RIGHTS ACT AND OUR DEMOCRACY

FairVote's Lakelaha Steele shares her family's story about the Voting Rights Act, and why we must defend it.

**LEARN MORE**

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**OUR REFORMS**

### WHAT IS RANKED CHOICE VOTING?

### WHAT IS PROPORTIONAL REPRESENTATION?

Iteratively remove all candidates with **the most number of voters who rank them last**, until there is a candidate with a majority of first-place votes.

If, at some stage of the removal process, all remaining candidates have the same number of voters who rank them last (so all candidates would be removed), then all remaining candidates are selected as winners.

| 7        | 5        | 4        | 3        |
|----------|----------|----------|----------|
| <i>a</i> | <i>b</i> | <i>d</i> | <i>c</i> |
| <i>b</i> | <i>c</i> | <i>b</i> | <i>d</i> |
| <i>c</i> | <i>d</i> | <i>c</i> | <i>a</i> |
| <i>d</i> | <i>a</i> | <i>a</i> | <i>b</i> |

| 7        | 5        | 4        | 3        |
|----------|----------|----------|----------|
| <i>a</i> | <i>b</i> | <i>d</i> | <i>c</i> |
| <i>b</i> | <i>c</i> | <i>b</i> | <i>d</i> |
| <i>c</i> | <i>d</i> | <i>c</i> | <i>a</i> |
| <i>d</i> | <i>a</i> | <i>a</i> | <i>b</i> |

Instant Runoff winners

| 7        | 5        | 4        | 3        |
|----------|----------|----------|----------|
| <i>a</i> | <i>b</i> | <i>d</i> | <i>c</i> |
| <i>b</i> | <i>c</i> | <i>b</i> | <i>d</i> |
| <i>c</i> | <i>d</i> | <i>c</i> | <i>a</i> |
| <i>d</i> | <i>a</i> | <i>a</i> | <i>b</i> |

Instant Runoff winners

|          |          |          |          |
|----------|----------|----------|----------|
| 7        | 5        | 4        | 3        |
| <i>a</i> | <i>b</i> | <i>d</i> | <i>c</i> |
| <i>b</i> | <i>c</i> | <i>b</i> | <i>d</i> |
| <i>c</i> | <i>d</i> | <i>c</i> | <i>a</i> |
| <i>d</i> | <i>a</i> | <i>a</i> | <i>b</i> |

Instant Runoff winners    *d*

| 7        | 5        | 4        | 3        |
|----------|----------|----------|----------|
| <i>a</i> | <i>b</i> | <i>d</i> | <i>c</i> |
| <i>b</i> | <i>c</i> | <i>b</i> | <i>d</i> |
| <i>c</i> | <i>d</i> | <i>c</i> | <i>a</i> |
| <i>d</i> | <i>a</i> | <i>a</i> | <i>b</i> |

Instant Runoff winners   *d*

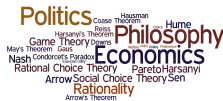
Coombs winners

|          |          |          |          |
|----------|----------|----------|----------|
| 7        | 5        | 4        | 3        |
| <i>a</i> | <i>b</i> | <i>d</i> | <i>c</i> |
| <i>b</i> | <i>c</i> | <i>b</i> | <i>d</i> |
| <i>c</i> | <i>d</i> | <i>c</i> | <i>a</i> |
| <i>d</i> | <i>a</i> | <i>a</i> | <i>b</i> |

Instant Runoff winners    *d*

Coombs winners            *b*

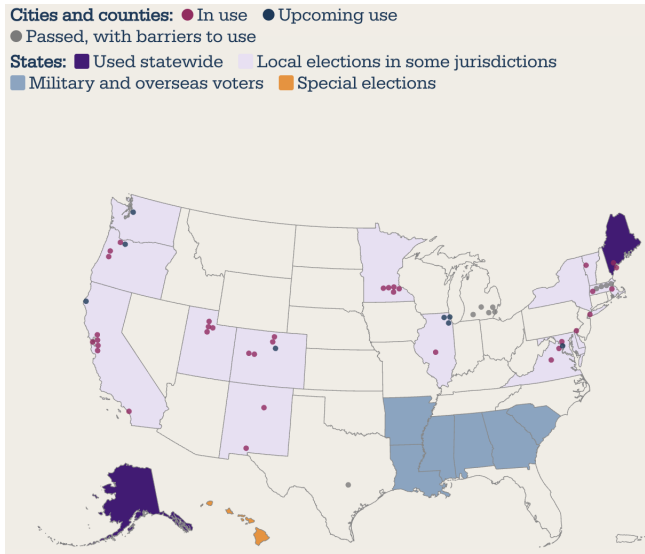
# Iterative Voting Methods



**Instant Runoff Voting (Ranked Choice):** Iteratively remove all candidates with the **fewest number of voters who rank them first**, until there is a candidate with a majority of first-place votes.

**Coombs:** Iteratively remove all candidates with the **most number of voters who rank them last**, until there is a candidate with a majority of first-place votes.

# Where is IRV being used?



[illegible]

The Instant Runoff Voting (IRV), also called Ranked Choice, winner is Peltola.

- 9 / 24

# 2022 Alaska Special General Election



## Round 1

| Candidate                       | Votes          | Percentage |
|---------------------------------|----------------|------------|
| Begich, Nick                    | 53,810         | 28.53%     |
| Palin, Sarah                    | 58,973         | 31.27%     |
| Peltola, Mary S.                | 75,799         | 40.19%     |
| <b>Continuing Ballots Total</b> | <b>188,582</b> |            |
| Blanks                          | 3,412          |            |
| Exhausted                       | 0              |            |
| Overvotes                       | 295            |            |
| Remainder Points                | 0              |            |
| <b>Non Transferable Total</b>   | <b>3,707</b>   |            |

**Begich, Nick is eliminated because the candidate had the least amount of votes.**

# 2022 Alaska Special General Election



Elimination transfer for candidate Begich, Nick.

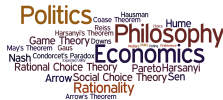
53810 ballots have been transferred in the following manner:

| Transferred from | Transferred to   | Ballots | Votes  |
|------------------|------------------|---------|--------|
| Begich, Nick     | Palin, Sarah     | 27053   | 27,053 |
| Begich, Nick     | Peltola, Mary S. | 15467   | 15,467 |
| Begich, Nick     | Exhausted        | 11243   | 11,243 |
| Begich, Nick     | Overvotes        | 47      | 47     |

| Candidate                       | Votes          | Percentage |
|---------------------------------|----------------|------------|
| Begich, Nick                    | 0              | 0.00%      |
| Palin, Sarah                    | 86,026         | 48.52%     |
| Peltola, Mary S.                | 91,266         | 51.48%     |
| <b>Continuing Ballots Total</b> | <b>177,292</b> |            |
| Blanks                          | 3,412          |            |
| Exhausted                       | 11,243         |            |
| Overvotes                       | 342            |            |
| Remainder Points                | 0              |            |
| <b>Non Transferable Total</b>   | <b>14,997</b>  |            |

12 / 24

# 2022 Alaska Special General Election



**Problem 1:** When comparing the winner Peltola with Begich, more voters rank Begich above Peltola than the other way around.

**So, in a head-to-head race, Begich is beats Peltola.**

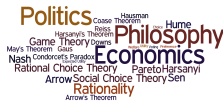
## So, in a head-to-head race, Begich is beats Peltola.

# Majority Preference, Margin, Condorcet Winners

| 40       | 35       | 25       |
|----------|----------|----------|
| <i>t</i> | <i>r</i> | <i>k</i> |
| <i>k</i> | <i>k</i> | <i>r</i> |
| <i>r</i> | <i>t</i> | <i>t</i> |

Why should candidate *k* win?

# Margin



Suppose that  $\mathbf{P}$  is an election (a record of the ballots submitted by the voters) and  $a$  and  $b$  are two candidates in  $\mathbf{P}$ .

The **margin of  $a$  over  $b$**  in  $\mathbf{P}$ , denoted  $\text{Margin}_{\mathbf{P}}(a, b)$ , is the number of voters that rank  $a$  above  $b$  in  $\mathbf{P}$  minus the number of voters that rank  $b$  above  $a$  in  $\mathbf{P}$ .

|     |     |     |
|-----|-----|-----|
| 40  | 35  | 25  |
| $t$ | $r$ | $k$ |
| $k$ | $k$ | $r$ |
| $r$ | $t$ | $t$ |

$$\text{Margin}_{\mathbf{P}}(t, k) = 40 - 60 = -20$$

$$\text{Margin}_{\mathbf{P}}(k, t) = 60 - 40 = 20$$

$$\text{Margin}_{\mathbf{P}}(k, r) = 30$$

$$\text{Margin}_{\mathbf{P}}(r, k) = -30$$

$$\text{Margin}_{\mathbf{P}}(t, r) = -20$$

$$\text{Margin}_{\mathbf{P}}(r, t) = 20$$

The **margin of  $a$  over  $b$**  in  $\mathbf{P}$ , denoted  $\text{Margin}_{\mathbf{P}}(a, b)$ , is the number of voters that rank  $a$  above  $b$  in  $\mathbf{P}$  minus the number of voters that rank  $b$  above  $a$  in  $\mathbf{P}$ .

$$Margin_{\mathbf{P}}(t, k) = 20$$

$$Margin_{\mathbf{P}}(k, t) = 20$$

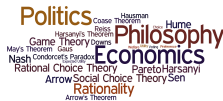
$$\text{Margin}_{\mathbf{P}}(k, r) = 65 - 35 = 30$$

$$\text{Margin}_{\mathbf{P}}(r, k) = 35 - 65 = -30$$

$$Margin_{\mathbf{P}}(t, r) = -20$$

$$Margin_{\mathbf{P}}(r, t) = 20$$

# Margin



Suppose that  $\mathbf{P}$  is an election (a record of the ballots submitted by the voters) and  $a$  and  $b$  are two candidates in  $\mathbf{P}$ .

The **margin of  $a$  over  $b$**  in  $\mathbf{P}$ , denoted  $\text{Margin}_{\mathbf{P}}(a, b)$ , is the number of voters that rank  $a$  above  $b$  in  $\mathbf{P}$  minus the number of voters that rank  $b$  above  $a$  in  $\mathbf{P}$ .

|     |     |     |
|-----|-----|-----|
| 40  | 35  | 25  |
| $t$ | $r$ | $k$ |
| $k$ | $k$ | $r$ |
| $r$ | $t$ | $t$ |

$$\text{Margin}_{\mathbf{P}}(t, k) = 20$$

$$\text{Margin}_{\mathbf{P}}(k, t) = 20$$

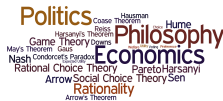
$$\text{Margin}_{\mathbf{P}}(k, r) = 30$$

$$\text{Margin}_{\mathbf{P}}(r, k) = -30$$

$$\text{Margin}_{\mathbf{P}}(t, r) = 40 - 60 = -20$$

$$\text{Margin}_{\mathbf{P}}(r, t) = 60 - 40 = 20$$

# Majority Graph



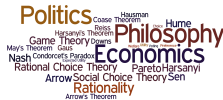
Suppose that  $\mathbf{P}$  is an election (a record of the ballots submitted by the voters) and  $a$  and  $b$  are two candidates in  $\mathbf{P}$ .

We say that  $a$  is **majority preferred** to  $b$  in  $\mathbf{P}$  when more voters rank  $a$  above  $b$  than rank  $b$  above  $a$ .

Alternatively,  $a$  is majority preferred to  $b$  when  $\text{Margin}_{\mathbf{P}}(a, b) > 0$ .

|     |     |     |                                    |     |     |                                    |
|-----|-----|-----|------------------------------------|-----|-----|------------------------------------|
| 40  | 35  | 25  | $\text{Margin}_{\mathbf{P}}(t, k)$ | $=$ | -20 |                                    |
| $t$ | $r$ | $k$ | $\text{Margin}_{\mathbf{P}}(k, t)$ | $=$ | 20  | ► $k$ is majority preferred to $t$ |
| $k$ | $k$ | $r$ | $\text{Margin}_{\mathbf{P}}(k, r)$ | $=$ | 30  | ► $k$ is majority preferred to $r$ |
| $r$ | $t$ | $t$ | $\text{Margin}_{\mathbf{P}}(r, k)$ | $=$ | -30 | ► $r$ is majority preferred to $t$ |
|     |     |     | $\text{Margin}_{\mathbf{P}}(t, r)$ | $=$ | -20 |                                    |
|     |     |     | $\text{Margin}_{\mathbf{P}}(r, t)$ | $=$ | 20  |                                    |

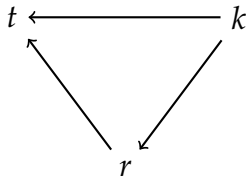
# Majority Graph



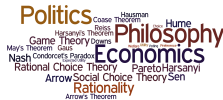
Suppose that  $\mathbf{P}$  is an election (a record of the ballots submitted by the voters) and  $a$  and  $b$  are two candidates in  $\mathbf{P}$ .

A **majority graph** is a diagram displaying all the candidates in the election with an arrow from candidate  $a$  to candidate  $b$  when  $a$  is majority preferred to  $b$  (i.e.,  $\text{Margin}_{\mathbf{P}}(a, b) > 0$ ).

|     |     |     |                                    |   |     |
|-----|-----|-----|------------------------------------|---|-----|
| 40  | 35  | 25  | $\text{Margin}_{\mathbf{P}}(t, k)$ | = | -20 |
| $t$ | $r$ | $k$ | $\text{Margin}_{\mathbf{P}}(k, t)$ | = | 20  |
| $k$ | $k$ | $r$ | $\text{Margin}_{\mathbf{P}}(k, r)$ | = | 30  |
| $r$ | $t$ | $t$ | $\text{Margin}_{\mathbf{P}}(r, k)$ | = | -30 |
|     |     |     | $\text{Margin}_{\mathbf{P}}(t, r)$ | = | -20 |
|     |     |     | $\text{Margin}_{\mathbf{P}}(r, t)$ | = | 20  |



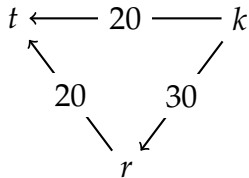
# Margin Graph



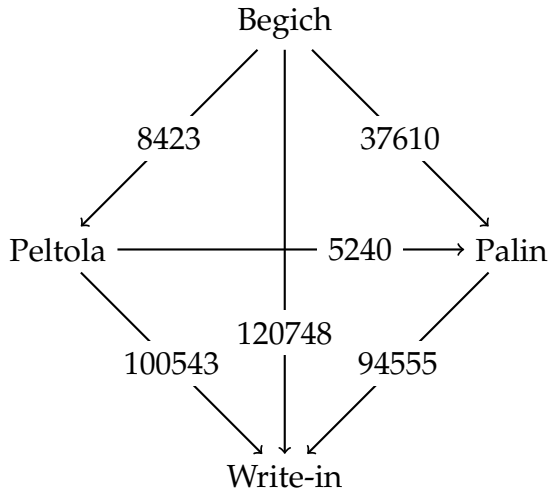
Suppose that  $\mathbf{P}$  is an election (a record of the ballots submitted by the voters) and  $a$  and  $b$  are two candidates in  $\mathbf{P}$ .

A **margin graph** is the majority graph in which all the arrows are labeled with the margins. That is, it is a diagram displaying all the candidates in the election with an arrow from candidate  $a$  to candidate  $b$  when  $a$  is majority preferred to  $b$ , and the arrow has the label  $\text{Margin}_{\mathbf{P}}(a, b)$ .

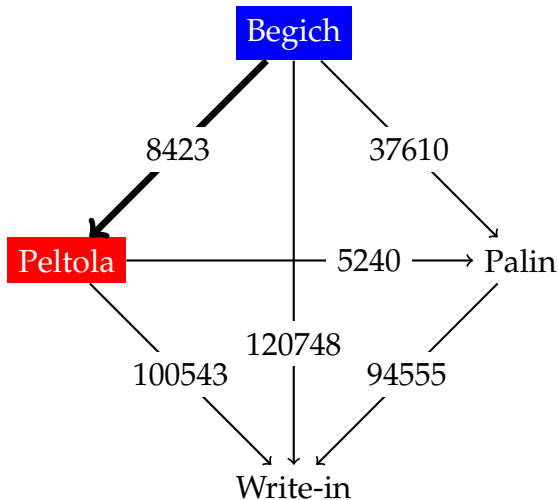
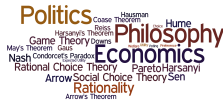
|     |     |     |                                    |     |       |
|-----|-----|-----|------------------------------------|-----|-------|
| 40  | 35  | 25  | $\text{Margin}_{\mathbf{P}}(t, k)$ | $=$ | $-20$ |
| $t$ | $r$ | $k$ | $\text{Margin}_{\mathbf{P}}(k, t)$ | $=$ | $20$  |
| $k$ | $k$ | $r$ | $\text{Margin}_{\mathbf{P}}(k, r)$ | $=$ | $30$  |
| $r$ | $t$ | $t$ | $\text{Margin}_{\mathbf{P}}(r, k)$ | $=$ | $-30$ |
|     |     |     | $\text{Margin}_{\mathbf{P}}(t, r)$ | $=$ | $-20$ |
|     |     |     | $\text{Margin}_{\mathbf{P}}(r, t)$ | $=$ | $20$  |



# 2022 Alaska Special General Election

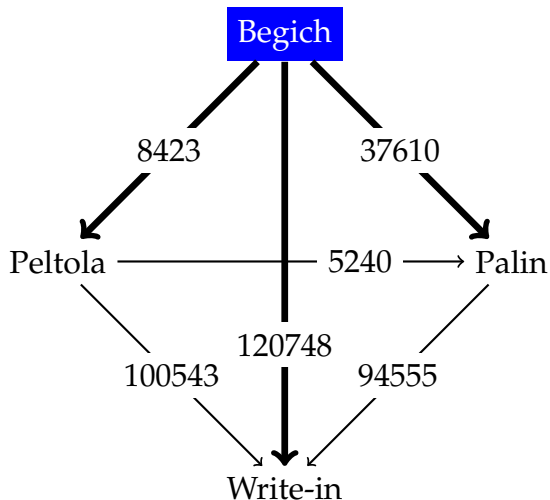
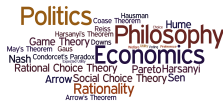


# 2022 Alaska Special General Election



- Problem 1: A majority of voters strictly prefer Begich to Peltola.

# 2022 Alaska Special General Election



- ▶ Problem 1: A majority of voters strictly prefer Begich to Peltola.
- ▶ Problem 2: In fact, Begich is majority preferred to every other candidate, but is not elected.
- ▶ See [consensus-choice.pacuit.org/results/alaska2022](https://consensus-choice.pacuit.org/results/alaska2022)

# Condorcet Winner



| 3        | 5        | 7        | 6        |
|----------|----------|----------|----------|
| <i>a</i> | <i>a</i> | <i>b</i> | <i>c</i> |
| <i>b</i> | <i>c</i> | <i>d</i> | <i>b</i> |
| <i>c</i> | <i>b</i> | <i>c</i> | <i>d</i> |
| <i>d</i> | <i>d</i> | <i>a</i> | <i>a</i> |

*a*

*b*

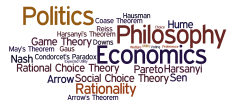
*d*

*c*

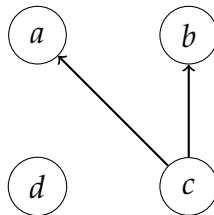
```

graph TD
    c((c)) --> a((a))
    b((b))
    d((d))
  
```

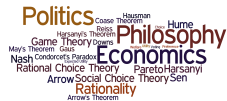
# Condorcet Winner



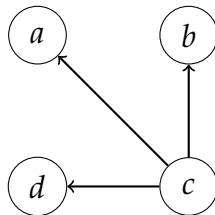
|          | 3        | 5        | 7        | 6        |
|----------|----------|----------|----------|----------|
| <i>a</i> | <i>a</i> | <i>a</i> | <i>b</i> | <i>c</i> |
| <i>b</i> | <i>b</i> | <i>c</i> | <i>d</i> | <i>b</i> |
| <i>c</i> | <i>c</i> | <i>b</i> | <i>c</i> | <i>d</i> |
| <i>d</i> | <i>d</i> | <i>d</i> | <i>a</i> | <i>a</i> |



# Condorcet Winner



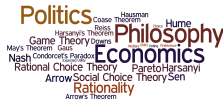
|          | 3        | 5        | 7        | 6        |
|----------|----------|----------|----------|----------|
| <i>a</i> | <i>a</i> | <i>a</i> | <i>b</i> | <i>c</i> |
| <i>b</i> | <i>b</i> | <i>c</i> | <i>d</i> | <i>b</i> |
| <i>c</i> | <i>c</i> | <i>b</i> | <i>c</i> | <i>d</i> |
| <i>d</i> | <i>d</i> | <i>d</i> | <i>a</i> | <i>a</i> |



```

graph TD
    a((a)) --> b((b))
    b --> a
    b --> d((d))
    d --> a
    c((c)) --> a
    c --> d
    style c fill:#0000FF,color:#FFFFFF
  
```

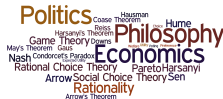
# Condorcet Winner



The **Condorcet winner** in a profile  $\mathbf{P}$  is a candidate  $x$  such that for all other candidates  $y$ ,  $\text{Margin}_{\mathbf{P}}(x, y) > 0$ .

A voting method is **Condorcet consistent**, if for all  $\mathbf{P}$ , if  $x$  is a Condorcet winner in  $\mathbf{P}$ , then  $x$  is the unique winner according to the voting method.

# Condorcet Loser



The **Condorcet loser** in a profile  $\mathbf{P}$  is a candidate  $x$  such that for all other candidates  $y$ ,  $\text{Margin}_{\mathbf{P}}(y, x) > 0$ .

Some voting methods (e.g., Plurality) may elect a Condorcet loser; while other voting methods (e.g., Borda) are guaranteed to never elect a Condorcet loser.

Borda, Plurality, Plurality with Runoff, Instant Runoff Voting, Coombs are **not** Condorcet consistent.

Can we find a voting method that is Condorcet consistent?

Borda, Plurality, Plurality with Runoff, Instant Runoff Voting, Coombs are **not** Condorcet consistent.

Can we find a voting method that is Condorcet consistent?

What about the method  $F_{cond}(\mathbf{P}) = \{a\}$  where  $a$  is the Condorcet winner in  $\mathbf{P}$ ?

# The Problem



| Voter 1 | Voter 2 | Voter 3 |
|---------|---------|---------|
| $a$     | $c$     | $b$     |
| $b$     | $a$     | $c$     |
| $c$     | $b$     | $a$     |

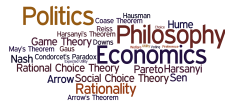
# The Problem



| Voter 1  | Voter 2  | Voter 3  |
|----------|----------|----------|
| <i>a</i> | <i>c</i> | <i>b</i> |
| <i>b</i> | <i>a</i> | <i>c</i> |
| <i>c</i> | <i>b</i> | <i>a</i> |

- Does the group prefer *a* over *b*?

# The Problem



| Voter 1  | Voter 2  | Voter 3  |
|----------|----------|----------|
| <i>a</i> | <i>c</i> | <i>b</i> |
| <i>b</i> | <i>a</i> | <i>c</i> |
| <i>c</i> | <i>b</i> | <i>a</i> |

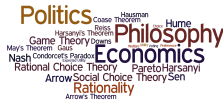
- Does the group prefer *a* over *b*? **Yes**

A 3x3 grid of colored squares with letters  $a$ ,  $b$ , and  $c$ . The grid is as follows:

|     |     |     |
|-----|-----|-----|
| $a$ | $c$ | $b$ |
| $b$ | $a$ | $c$ |
| $c$ | $b$ | $a$ |

- Does the group prefer  $a$  over  $b$ ? **Yes**
- Does the group prefer  $b$  over  $c$ ? **Yes**

# The Problem

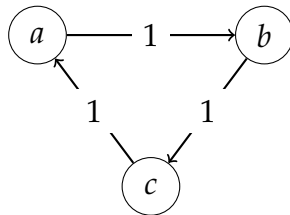


| Voter 1  | Voter 2  | Voter 3  |
|----------|----------|----------|
| <i>a</i> | <i>c</i> | <i>b</i> |
| <i>b</i> | <i>a</i> | <i>c</i> |
| <i>c</i> | <i>b</i> | <i>a</i> |

- ▶ Does the group prefer *a* over *b*? Yes
- ▶ Does the group prefer *b* over *c*? Yes
- ▶ Does the group prefer *a* over *c*? No

# Majority Cycle Example

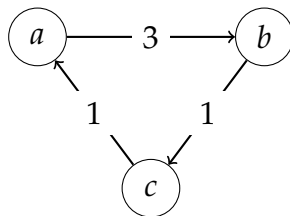
| 1        | 1        | 1        |
|----------|----------|----------|
| <i>a</i> | <i>c</i> | <i>b</i> |
| <i>b</i> | <i>a</i> | <i>c</i> |
| <i>c</i> | <i>b</i> | <i>a</i> |





# Majority Cycle Example

| 2        | 2        | 1        |
|----------|----------|----------|
| <i>a</i> | <i>c</i> | <i>b</i> |
| <i>b</i> | <i>a</i> | <i>c</i> |
| <i>c</i> | <i>b</i> | <i>a</i> |



# Not a Majority Cycle

| 1        | 5        | 3        |
|----------|----------|----------|
| <i>a</i> | <i>c</i> | <i>b</i> |
| <i>b</i> | <i>a</i> | <i>c</i> |
| <i>c</i> | <i>b</i> | <i>a</i> |

