PHPE 400
Individual and Group Decision Making

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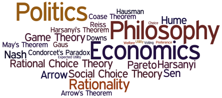
Rational choice



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Mathematical background: Relations

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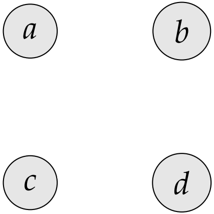
A **relation** on X is a set of **ordered pairs** from X.

That is, if R is a relation on X, then $R \subseteq X \times X$.

Mathematical background: Relations

Example: $X = \{a, b, c, d\}$, $R = \{(a,a), (b,a), (c,d), (a,c), (d,d)\}$



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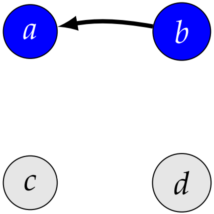

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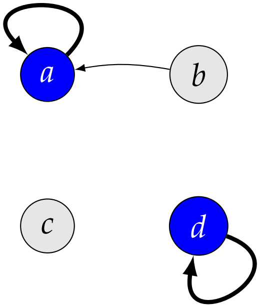

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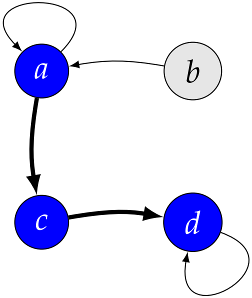

$a\ R\ a$
$b\ R\ a$
$c\ R\ d$
$a\ R\ c$
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Strict Preference

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If P represents the decision maker's strict preference and $x \; P \; y$ (i.e., the decision maker strictly prefers x to y), then the decision maker would pay some non-zero amount money to trade y for x.

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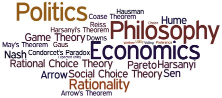

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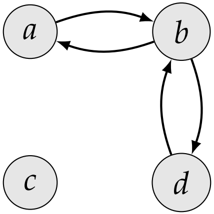
Can *any* relation on X represent a strict preference for a decision maker?

Symmetric/Asymmetric Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Symmetric relation: for all $x, y \in X$, if $x \; R \; y$, then $y \; R \; x$

Asymmetric relation: for all $x, y \in X$, if $x \; R \; y$, then not-$y \; R \; x$



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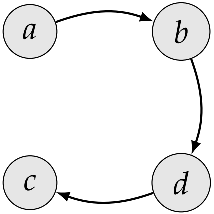

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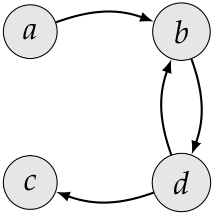

not symmetric and not asymmetric

Strict Preference

A decision maker's strict preference over a set X is represented as a relation $P \subseteq X \times X$.

The underlying idea is that if P represents the decision maker's strict preference and $x\ P\ y$ (i.e., the decision maker strictly prefers x to y), then the decision maker would pay some non-zero amount money to trade y for x.

Assumption: P is asymmetric (for all $x, y \in X$, if $x\ P\ y$, then it is not the case that $y\ P\ x$, written not-$y\ P\ x$).

Indifference/Incommensurable

Suppose that P is an asymmetric relation on X (interpreted as a decision maker's strict preference). Suppose that $x, y \in X$ with not-$x \; P \; y$ and not-$y \; P \; x$.

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There are two reasons why this might hold:

1. The decision maker is *indifferent* between x and y.
   In this case, we write $x \; I \; y$.
2. The decision maker *cannot compare* x and y.
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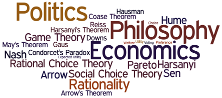

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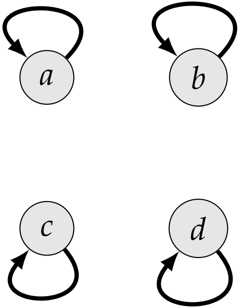
What properties should I and N satisfy?

Reflexive Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Reflexive relation: for all $x \in X$, $x\ R\ x$



Representing Preferences

Let X be a set of outcomes. A decision maker's *preference* over X is represented by *relations* on X:

- $P \subseteq X \times X$ where $a \ P \ b$ means that the decision maker *strictly prefers* a to b.

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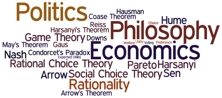

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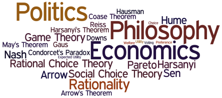
- $N \subseteq X \times X$ where $a \, N \, b$ means that the decision maker *cannot compare* a and b.

Preferences - Minimal Constraints

A decision maker's preferences on X is represented by three relations $P \subseteq X \times X$, $I \subseteq X \times X$ and $N \subseteq X \times X$ satisfying the following minimal constraints:

1. For all $x, y \in X$, exactly one of $x\ P\ y$, $y\ P\ x$, $x\ I\ y$ and $x\ N\ y$ is true.
2. P is asymmetric
3. I is reflexive and symmetric.
4. N is symmetric.

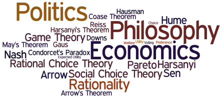
Rational Preferences



An individual's preferences are **rational** when they satisfy two additional constraints:

1. transitivity
2. completeness

Transitive Relations



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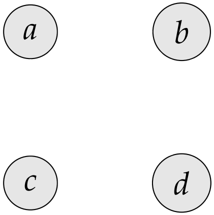

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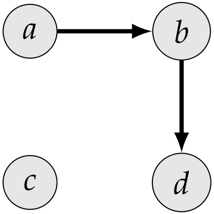

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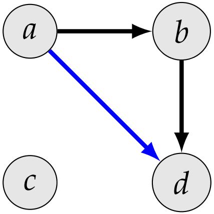

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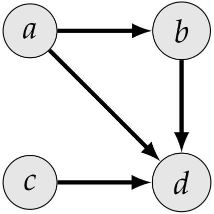

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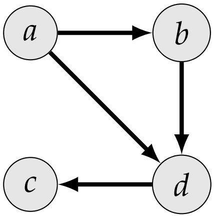

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