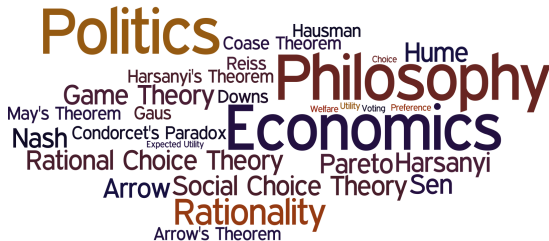


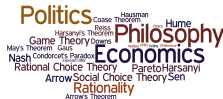
# PHPE 400

## Individual and Group Decision Making

Eric Pacuit  
University of Maryland  
[pacuit.org](http://pacuit.org)



# First Steps



1. Make sure you are signed up and can login to [Piazza](#) (available on the course website)
2. Sign up for [Tophat](#) with join code 209898 (available on the course website)
3. Read the course policies (<https://phpe400.info/policies>) and syllabus (<https://umd.instructure.com/courses/1390147/assignments/syllabus>).

# Grading



Participation 30%

Problem Sets 40%

Midterm 15%

Final Exam 15%

<https://umd.instructure.com/courses/1390147>

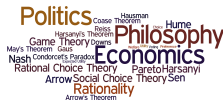
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[https://umd.instructure.com/courses/1390147/external\\_tools/81891](https://umd.instructure.com/courses/1390147/external_tools/81891)

<https://umd.instructure.com/courses/1390147/modules>  
<https://notes.phpe400.info>

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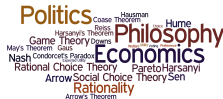
# Practicalities: Math



The course is completely self-contained, but it does require that you become comfortable with some mathematical notation.

- For example, sets  $X = \{a, b, c\}$ , subset of  $X \subseteq Y$ , element of  $x \in X$ , cross-product  $X \times Y = \{(x, y) \mid x \in X, y \in Y\}$ , relations  $R \subseteq X \times X$ , functions  $f : X \rightarrow Y, \dots$

# Practicalities: Math



- ▶ Ask questions, especially about notation that you do not understand (no matter how trivial).
- ▶ The participation questions are designed, in part, to make sure you understand the mathematical notation.
- ▶ It is important to use the proper notation on the problem sets and the exams (otherwise we won't understand your answers).
- ▶ Attend the discussion sections.

# Practicalities: Math



Economic models consist of clearly stated assumptions and behavioral mechanisms. As such, they lend themselves to the language of mathematics. Flip the pages of any academic journal in economics and you will encounter a nearly endless stream of equations and Greek symbols...



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(Rodrik, pp. 22-23)

D. Rodrik (2015). *Economic Rules: The Rights and Wrongs of the Dismal Science*. W. W. Norton.

# What is this course about?



1. What principles determine whether individual and group decisions are **rational**?

Politics Philosophy Economics

Game Theory Rational Choice Theory Arrow's Theorem

Nash Condorcet's Paradox Harsanyi's Theorem Pareto Harsanyi

May's Theorem Gaus Rationality Social Choice Theory Sen

Arrow's Theorem Rationality Arrow's Theorem

Hausman Reiss Coase Theorem Hume

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# (Useful?) Assumptions

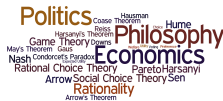


In truth, simple models of the type that economists construct are absolutely essential to understanding the workings of society. Their simplicity, formalism, and neglect of many facets of the real world are precisely what makes them valuable. These are a feature, not a bug.

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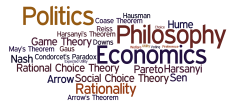


In truth, simple models of the type that economists construct are absolutely essential to understanding the workings of society. Their simplicity, formalism, and neglect of many facets of the real world are precisely what makes them valuable. These are a feature, not a bug.

What makes a model useful is that it captures an aspect of reality. What makes it indispensable, when used well, is that **it captures the most relevant aspect of reality in a given context.** (p. 11, Rodrik)

D. Rodrik (2015). *Economics Rules: The Rights and Wrongs of the Dismal Science*. W. W. Norton.

# Topics



## Topics

## Decision Theory: How should individuals make decisions under uncertainty?



# Topics

Politics  
Coase Theorem  
Hausman  
Hume  
Philosophy  
Game Theory  
Harsanyi's Theorem  
Downs  
May's Theorem  
Gaus  
Nash  
Condorcet's Paradox  
Rational Choice Theory  
Arrow  
Social Choice Theory  
Pareto  
Harsanyi  
Sen  
Rationality  
Arrow's Theorem  
Economics

**Decision Theory:** How should individuals make decisions under uncertainty?

**Game Theory:** How should individuals strategize in interactive situations?





# Topics



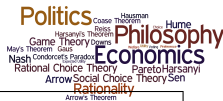
**Decision Theory:** How should individuals make decisions under uncertainty?

**Game Theory:** How should individuals strategize in interactive situations?

**Social Choice Theory:** How should a group aggregate individual opinions to reach a collective decision?



# Tentative Schedule



Date	Topic
9/3	Introduction, Rational preferences
9/8 9/10	Rational Preferences
9/15 9/17	Expected utility theory
9/22 9/24	Expected utility theory Evaluating rational choice axioms
9/29 10/1	Evaluating rational choice axioms Decision theory
10/6 10/8	Decision theory Introduction to game theory
<b>10/13</b>	<b>No Class: Fall Break</b>
10/15	Introduction to game theory
10/20 10/22	Introduction to game theory <b>Midterm Exam</b>

Date	Topic
10/27 10/29	Voting
11/3 11/5	Voting
11/10 11/12	Topics in social choice theory
11/17 11/19	Topics in social choice theory
11/24 <b>11/26</b>	Topics in social choice theory <b>No Class: Thanksgiving Break</b>
12/1 12/3	Aggregating utilities
12/8 12/10	Aggregating utilities
12/19	<b>Final Exam</b>

# Rational Preferences

# Simple Choice Model

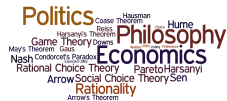
Menu



Politics  
Coase  
Hausman  
Theorem  
Hume  
Philosophy  
Game Theory  
Harsanyi's Theorem  
May's Theorem  
Nash  
Condorcet's Paradox  
Rational Choice Theory  
Arrow  
Social Choice  
Pareto  
Harsanyi  
Theory  
Sen  
Rationality  
Arrow's Theorem

Three glasses are shown side-by-side. The first is a wine glass filled with red wine. The second is a beer glass filled with beer and a head of foam. The third is a tall glass filled with a yellow beverage, ice, and lemon slices, garnished with a sprig of rosemary.

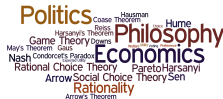
# Simple Choice Model



Rational Choice?



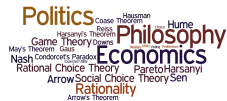
# Simple Choice Model



The concept of “preference” is central to economic theory. Economists typically take preferences to be predetermined or “given” facts about individuals and, for their purposes, not in need of explanation or subject to substantive appraisal. Economic analyses begin with an individual’s preferences, whatever that may be.

(p. 56, Hausman, McPherson and Satz)

# Simple Choice Model



Rational Choice?



Preference



$P$



$P$





# Simple Choice Model



Rational Choice



Preference



$P$



$P$



# Simple Choice Model



Irrational Choice



Preference



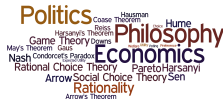
$P$



$P$

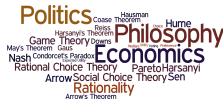


# Preferences *and* Beliefs



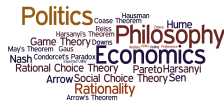
- **Option uncertainty:** What type of wine is it? Is the red wine sweet or dry? Is the white wine spoiled? Is the lemonade very sugary? . . .

# Preferences *and* Beliefs



- ▶ **Option uncertainty:** What type of wine is it? Is the red wine sweet or dry? Is the white wine spoiled? Is the lemonade very sugary? . . .
- ▶ **Context:** What are we having to eat? What time of day is it? How many drinks have you had? Are you driving home? Are there other drink choices that are available (e.g., a beer or a soda)? . . .

# Preferences



Preferring or choosing  $x$  is different than “liking”  $x$  or “having a taste for  $x$ ”:  
one can prefer  $x$  to  $y$  but *dislike* both options

Preferences are always understood as *comparative*: “preference” is more like  
“bigger” than “big”

# Concepts of *preference*



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3. *Choice ranking*: In a restaurant, when asked “do you prefer red wine or white wine”, the waiter wants to know which option I choose.
4. *Comparative evaluation*: I prefer candidate *A* over candidate *B* means “I judge *A* to be *superior* to *B*”. This can be *partial* (ranking with respect to some criterion) or *total* (with respect to every relevant consideration).

A word cloud visualization showing the frequency of various economic theories and concepts. The words are arranged in a circular pattern, with 'Economics' being the largest and most central word. Other prominent words include 'Philosophy', 'Politics', 'Rationality', 'Arrow's Theorem', 'Social Choice Theory', 'Pareto', 'Harsanyi', 'Nash', 'Game Theory', 'Coase Theorem', 'Hausman', 'Reiss', 'May's Theorem', 'Gauss', 'Condorcet's Paradox', 'Rational Choice Theory', 'Theory Sen', and 'Arrow's Theorem'.

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# Rational choice



A decision maker chooses rationally if her preferences are rational and there is nothing available that the decision maker prefers to what she chooses.

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# Mathematical background: Relations



Suppose that  $X$  is a set.

An **ordered pair** of elements from  $X$  is  $(a, b)$  where  $a \in X$  is the first component and  $b \in X$  is the second component.

# Mathematical background: Relations

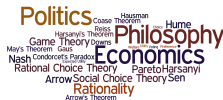


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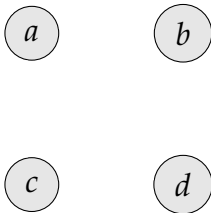
A **relation** on  $X$  is a set of **ordered pairs** from  $X$ .

That is, if  $R$  is a relation on  $X$ , then  $R \subseteq X \times X$ .

# Mathematical background: Relations



Example:  $X = \{a, b, c, d\}$ ,  $R = \{(a, a), (b, a), (c, d), (a, c), (d, d)\}$





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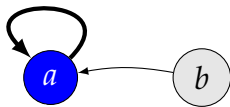


$b R a$



# Mathematical background: Relations

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$a R a$

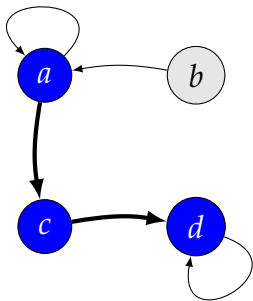
$b R a$



$d R d$

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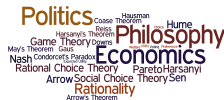
$b R a$

$c R d$

$a R c$

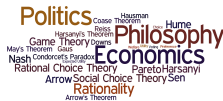
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# Strict Preference



A decision maker's strict preference over a set  $X$  is represented as a relation  $P \subseteq X \times X$ .

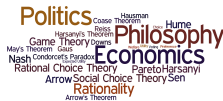
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A decision maker's strict preference over a set  $X$  is represented as a relation  $P \subseteq X \times X$ .

If  $P$  represents the decision maker's strict preference and  $x P y$  (i.e., the decision maker strictly prefers  $x$  to  $y$ ), then the decision maker would pay some non-zero amount money to trade  $y$  for  $x$ .

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Can *any* relation on  $X$  represent a strict preference for a decision maker?

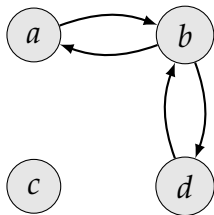
# Symmetric/Asymmetric Relations



Suppose that  $X$  is a set and  $R \subseteq X \times X$  is a relation.

**Symmetric relation:** for all  $x, y \in X$ , if  $x R y$ , then  $y R x$

**Asymmetric relation:** for all  $x, y \in X$ , if  $x R y$ , then not- $y R x$



symmetric but not asymmetric

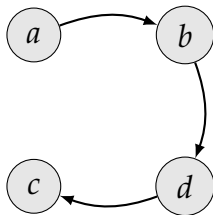
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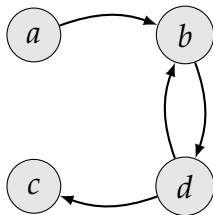
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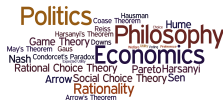
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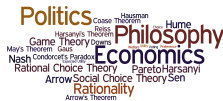


A decision maker's strict preference over a set  $X$  is represented as a relation  $P \subseteq X \times X$ .

The underlying idea is that if  $P$  represents the decision maker's strict preference and  $x P y$  (i.e., the decision maker strictly prefers  $x$  to  $y$ ), then the decision maker would pay some non-zero amount money to trade  $y$  for  $x$ .

**Assumption:**  $P$  is asymmetric (for all  $x, y \in X$ , if  $x P y$ , then it is not the case that  $y P x$ , written not- $y P x$ ).

# Indifference/Incommensurable



Suppose that  $P$  is an asymmetric relation on  $X$  (interpreted as a decision maker's strict preference). Suppose that  $x, y \in X$  with  $\text{not-}x P y$  and  $\text{not-}y P x$ .

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There are two reasons why this might hold:

1. The decision maker is *indifferent* between  $x$  and  $y$ .  
In this case, we write  $x I y$ .
2. The decision maker *cannot compare*  $x$  and  $y$ .  
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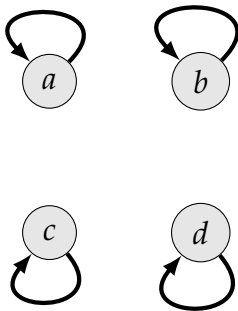
What properties should  $I$  and  $N$  satisfy?

# Reflexive Relations



Suppose that  $X$  is a set and  $R \subseteq X \times X$  is a relation.

**Reflexive relation:** for all  $x \in X$ ,  $x R x$



# Representing Preferences



Let  $X$  be a set of outcomes. A decision maker's *preference* over  $X$  is represented by *relations* on  $X$ :

- $P \subseteq X \times X$  where  $a P b$  means that the decision maker *strictly prefers*  $a$  to  $b$ .

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- ▶  $N \subseteq X \times X$  where  $a N b$  means that the decision maker *cannot compare*  $a$  and  $b$ .

# Preferences - Minimal Constraints



A decision maker's preferences on  $X$  is represented by three relations  $P \subseteq X \times X$ ,  $I \subseteq X \times X$  and  $N \subseteq X \times X$  satisfying the following minimal constraints:

1. For all  $x, y \in X$ , exactly one of  $x P y$ ,  $y P x$ ,  $x I y$  and  $x N y$  is true.
2.  $P$  is asymmetric
3.  $I$  is reflexive and symmetric.
4.  $N$  is symmetric.