PHPE 400
Individual and Group Decision Making

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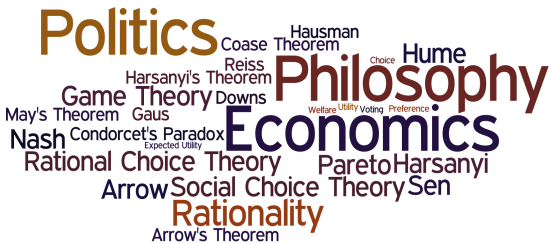


Collective decision making

Voter 1  >  > 

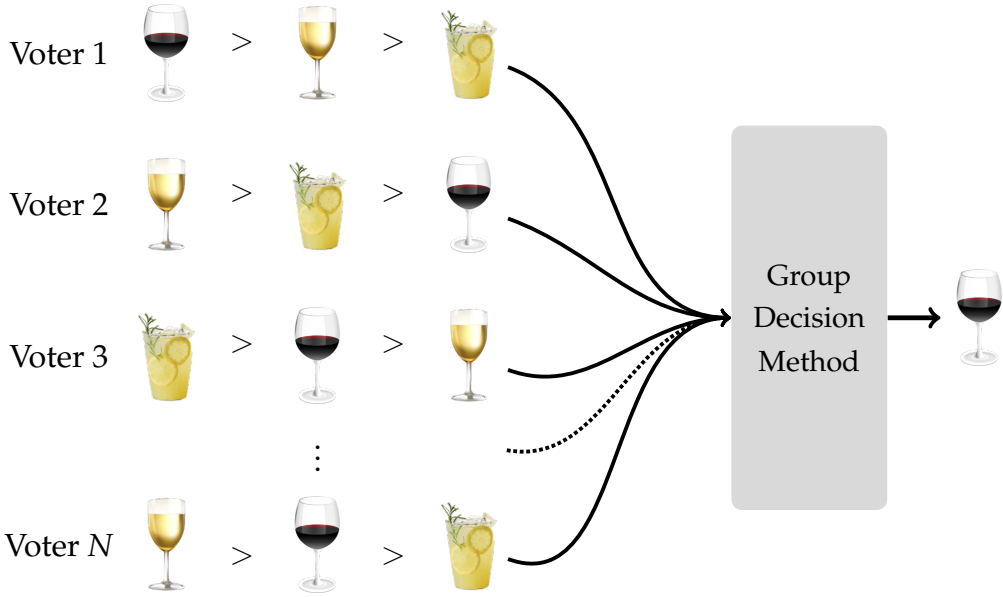
Voter 2  >  > 

Voter 3  >  > 

⋮

Voter N  >  > 


Voter 1
>
>
Voter 2
>
>
Voter 3
>
>
Voter N
>
>
Group
Decision
Method


Voter 1
>
>
Voter 2
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Voter 3
>
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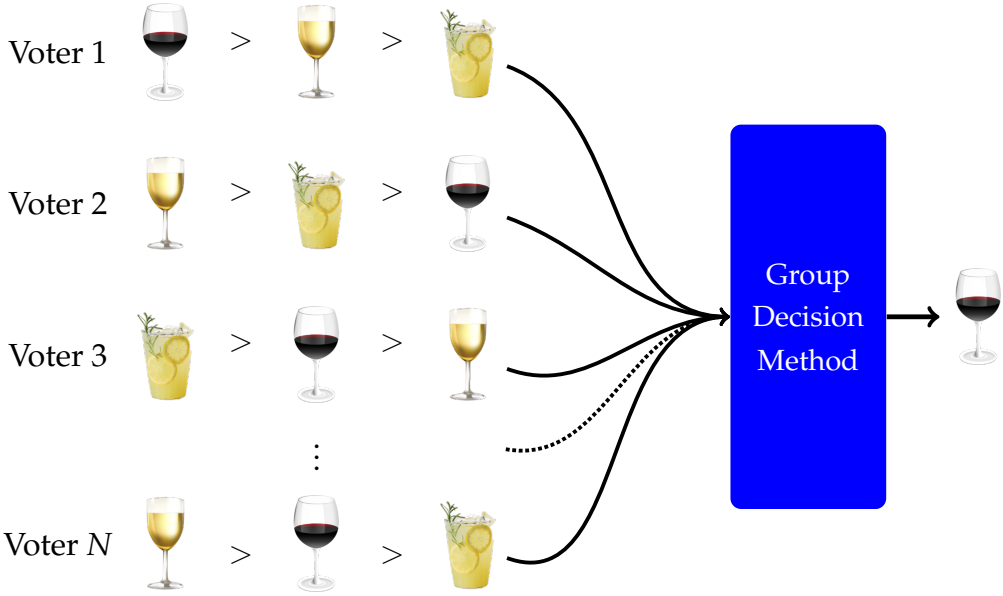
Group
Decision
Method


Voter 1
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Voter 2
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Voter 3
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Voter N
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Group
Decision
Method

Which candidate *should* be chosen?

40	35	25
t	r	k
k	k	r
r	t	t

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- No candidate is the **majority winner**.
  No candidate has a **majority** of 1st place votes.

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- The **Plurality** winner is t
  The plurality is the candidate that is ranked first by the most voters.

Which candidate *should* be chosen?

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- No candidate is the **majority winner**.
  No candidate has a **majority** of 1st place votes.
- The **Plurality** winner is t
  The plurality is the candidate that is ranked first by the most voters.
- The **Instant Runoff** winner is r
  After k is removed since it is ranked first by the fewest number of voters, candidate r is the majority winner.

40	35	25
t	r	k
k	k	r
r	t	t

What about candidate k?

Margin

Suppose that **P** is an election (a record of the ballots submitted by the voters) and a and b are two candidates in **P**.

The **margin of** a **over** b in **P**, denoted $Margin_{\mathbf{P}}(a, b)$, is the number of voters that rank a above b in **P** minus the number of voters that rank b above a in **P**.

40	35	25
t	r	k
k	k	r
r	t	t

$$
\begin{aligned}
Margin_{\mathbf{P}}(t,k) &= 40-60=-20\\
Margin_{\mathbf{P}}(k,t) &= 60-40=20\\
Margin_{\mathbf{P}}(k,r) &= 30\\
Margin_{\mathbf{P}}(r,k) &= -30\\
Margin_{\mathbf{P}}(t,r) &= -20\\
Margin_{\mathbf{P}}(r,t) &= 20
\end{aligned}
$$

Margin

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40	35	25
t	r	k
k	k	r
r	t	t

$$
\begin{aligned}
Margin_{\mathbf{P}}(t,k) &= 20 \\
Margin_{\mathbf{P}}(k,t) &= 20 \\
Margin_{\mathbf{P}}(k,r) &= 65-35=30 \\
Margin_{\mathbf{P}}(r,k) &= 35-65=-30 \\
Margin_{\mathbf{P}}(t,r) &= -20 \\
Margin_{\mathbf{P}}(r,t) &= 20
\end{aligned}
$$

Margin

Suppose that **P** is an election (a record of the ballots submitted by the voters) and a and b are two candidates in **P**.

The **margin of** a **over** b in **P**, denoted $Margin_{\mathbf{P}}(a, b)$, is the number of voters that rank a above b in **P** minus the number of voters that rank b above a in **P**.

40	35	25
t	r	k
k	k	r
r	t	t

$$
\begin{aligned}
Margin_{\mathbf{P}}(t,k) &= 20 \\
Margin_{\mathbf{P}}(k,t) &= 20 \\
Margin_{\mathbf{P}}(k,r) &= 30 \\
Margin_{\mathbf{P}}(r,k) &= -30 \\
Margin_{\mathbf{P}}(t,r) &= 40 - 60 = -20 \\
Margin_{\mathbf{P}}(r,t) &= 60 - 40 = 20
\end{aligned}
$$

Majority Graph

Suppose that **P** is an election (a record of the ballots submitted by the voters) and a and b are two candidates in **P**.

We say that a is **majority preferred** to b in **P** when more voters rank a above b than rank b above a.

Alternatively, a is majority preferred to b when $Margin_{\mathbf{P}}(a, b) > 0$.

40	35	25
t	r	k
k	k	r
r	t	t

$$
\begin{aligned}
Margin_{\mathbf{P}}(t,k) &= -20 \\
Margin_{\mathbf{P}}(k,t) &= 20 \\
Margin_{\mathbf{P}}(k,r) &= 30 \\
Margin_{\mathbf{P}}(r,k) &= -30 \\
Margin_{\mathbf{P}}(t,r) &= -20 \\
Margin_{\mathbf{P}}(r,t) &= 20
\end{aligned}
$$

- k is majority preferred to t
- k is majority preferred to r
- r is majority preferred to t

Majority Graph

Suppose that **P** is an election (a record of the ballots submitted by the voters) and a and b are two candidates in **P**.

A **majority graph** is a diagram displaying all the candidates in the election with an arrow from candidate a to candidate b when a is majority preferred to b (i.e., $Margin_{\mathbf{P}}(a,b) > 0$).

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k	k	r
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Margin_{\mathbf{P}}(t,r) &= -20 \\
Margin_{\mathbf{P}}(r,t) &= 20
\end{aligned}
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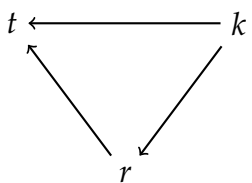

Margin Graph

Suppose that **P** is an election (a record of the ballots submitted by the voters) and a and b are two candidates in **P**.

A **margin graph** is the majority graph in which all the arrows are labeled with the margins. That is, it is a diagram displaying all the candidates in the election with an arrow from candidate a to candidate b when a is majority preferred to b, and the arrow has the label $Margin_{\mathbf{P}}(a, b)$.

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\begin{aligned}
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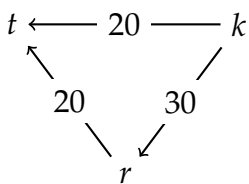
Margin_{\mathbf{P}}(t,r) &= -20 \\
Margin_{\mathbf{P}}(r,t) &= 20
\end{aligned}
$$



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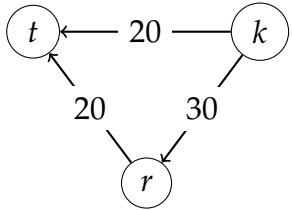

Plurality winner	t
Instant Runoff winner	r
Condorcet winner	k
Condorcet loser	t

Suppose that **P** is an election (a record of the ballots submitted by the voters).

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When more voters rank candidate A above candidate B than the other way around, the voters *favor* A over B in **P**.

In most elections, the voting public favors one candidate over all others. This candidate is called the *Condorcet winner*.

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When more voters rank candidate A above candidate B than the other way around, the voters *favor* A over B in **P**.

In most elections, the voting public favors one candidate over all others. This candidate is called the *Condorcet winner*.

- A candidate a is the **Condorcet winner** in **P** when a is majority preferred to every other candidate in **P**.

  Alternatively, For all candidates b other than a, $Margin_{\mathbf{P}}(a, b) > 0$.

  Alternatively, For all candidates b other than a, there is an arrow from a to b in the majority (margin) graph for **P**