# PHPE 400 <br> Individual and Group Decision Making 

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# Collective decision making 








## Which candidate should be chosen?

 Nash Condorcet's Paradox ECO Con ParetoHarsanyi
Rational Choice Theory
ArrowSocial Choice TheorySen Arrow Social Choice
Rationality
arrows theocrem

$$
\begin{array}{ccc}
40 & 35 & 25 \\
\hline t & r & k \\
k & k & r \\
r & t & t
\end{array}
$$

## Which candidate should be chosen?

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| 40 | 35 | 25 |
| :---: | :---: | :---: |
| $t$ | $r$ | $k$ |
| $k$ | $k$ | $r$ |
| $r$ | $t$ | $t$ |

- No candidate is the majority winner. No candidate has a majority of 1st place votes.


## Which candidate should be chosen?

Politicscasan fumi fum Mas sime theor ame Nash Consorcets Paradox
Rational Choice Theory ParetoHarsanyi

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- No candidate is the majority winner. No candidate has a majority of 1st place votes.
- The Plurality winner is $t$

The plurality is the candidate that is ranked first by the most voters.

## Which candidate should be chosen?

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- The Plurality winner is $t$

The plurality is the candidate that is ranked first by the most voters.

- The Instant Runoff winner is $r$

After $k$ is removed since it is ranked first by the fewest number of voters, candidate $r$ is the majority winner.


What about candidate $k$ ?

## Margin

Suppose that $\mathbf{P}$ is an election (a record of the ballots submitted by the voters) and $a$ and $b$ are two candidates in $\mathbf{P}$.

The margin of $a$ over $b$ in $\mathbf{P}$, denoted $\operatorname{Margin}_{\mathbf{P}}(a, b)$, is the number of voters that rank $a$ above $b$ in $\mathbf{P}$ minus the number of voters that rank $b$ above $a$ in $\mathbf{P}$.


$$
\begin{aligned}
\operatorname{Margin}_{\mathbf{P}}(t, k) & =40-60=-20 \\
\operatorname{Margin}_{\mathbf{P}}(k, t) & =60-40=20 \\
\operatorname{Margin}_{\mathbf{P}}(k, r) & =30 \\
\operatorname{Margin}_{\mathbf{P}}(r, k) & =-30 \\
\operatorname{Margin}_{\mathbf{P}}(t, r) & =-20 \\
\operatorname{Margin}_{\mathbf{P}}(r, t) & =20
\end{aligned}
$$

## Margin

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$$
\begin{aligned}
& \operatorname{Margin}_{\mathbf{P}}(t, k)=20 \\
& \operatorname{Margin}_{\mathbf{P}}(k, t)=20 \\
& \operatorname{Margin}_{\mathbf{p}}(k, r)=65-35=30 \\
& \operatorname{Margin}_{\mathrm{P}}(r, k)=35-65=-30 \\
& \operatorname{Margin}_{\mathbf{P}}(t, r)=-20 \\
& \operatorname{Margin}_{\mathbf{P}}(r, t)=20
\end{aligned}
$$

## Margin

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$$
\begin{aligned}
& \operatorname{Margin}_{\mathbf{P}}(t, k)=20 \\
& \operatorname{Margin}_{\mathbf{P}}(k, t)=20 \\
& \operatorname{Margin}_{\mathbf{P}}(k, r)=30 \\
& \operatorname{Margin} \mathbf{P}^{\mathbf{P}}(r, k)=-30 \\
& \operatorname{Margin}_{\mathbf{P}}(t, r)
\end{aligned}=40-60=-20
$$

## Majority Graph

Suppose that $\mathbf{P}$ is an election (a record of the ballots submitted by the voters) and $a$ and $b$ are two candidates in $\mathbf{P}$.
We say that $a$ is majority preferred to $b$ in $\mathbf{P}$ when more voters rank $a$ above $b$ than rank $b$ above $a$.

Alternatively, $a$ is majority preferred to $b$ when $\operatorname{Margin}_{\mathbf{P}}(a, b)>0$.

$$
\begin{aligned}
& \operatorname{Margin}_{\mathbf{P}}(t, k)=-20 \\
& \begin{array}{ccc}
40 & 35 & 25 \\
\hline t & r & k
\end{array} \\
& k \quad k \quad \operatorname{Margin}_{\mathbf{P}}(r, k)=-30 \\
& r t \quad t \quad \operatorname{Margin}_{\mathbf{P}}(t, r)=-20 \\
& \operatorname{Margin}_{\mathbf{P}}(r, t)=20 \\
& \text { - } k \text { is majority preferred to } t \\
& \text { - } k \text { is majority preferred to } r \\
& \text { - } r \text { is majority preferred to } t
\end{aligned}
$$

## Majority Graph

Suppose that $\mathbf{P}$ is an election (a record of the ballots submitted by the voters) and $a$ and $b$ are two candidates in $\mathbf{P}$.
A majority graph is a diagram displaying all the candidates in the election with an arrow from candidate $a$ to candidate $b$ when $a$ is majority preferred to $b$ (i.e., $\operatorname{Margin}_{\mathbf{P}}(a, b)>0$ ).

$$
\begin{aligned}
& \operatorname{Margin}_{\mathbf{p}}(t, k)=-20 \\
& \begin{array}{ccc}
40 & 35 & 25
\end{array} \begin{array}{cc}
\operatorname{Margin}_{\mathbf{P}}(k, t) & =20 \\
\hline t & r \\
\operatorname{Margin}_{\mathbf{P}}(k, r) & =30
\end{array} \\
& k \quad \begin{aligned}
& \operatorname{Margin}_{\mathbf{P}}(r, k) \\
= & -30
\end{aligned} \\
& r t \quad t \quad \operatorname{Margin}_{\mathbf{P}}(t, r)=-20 \\
& \operatorname{Margin}_{\mathbf{P}}(r, t)=20
\end{aligned}
$$



## Margin Graph

Suppose that $\mathbf{P}$ is an election (a record of the ballots submitted by the voters) and $a$ and $b$ are two candidates in $\mathbf{P}$.

A margin graph is the majority graph in which all the arrows are labeled with the margins. That is, it is a diagram displaying all the candidates in the election with an arrow from candidate $a$ to candidate $b$ when $a$ is majority preferred to $b$, and the arrow has the label $\operatorname{Margin}_{\mathbf{P}}(a, b)$.

$$
\begin{aligned}
& \operatorname{Margin}_{\mathbf{P}}(t, k)=-20 \\
& \begin{array}{ccc}
40 & 35 & 25
\end{array} \begin{array}{c}
\operatorname{Margin}_{\mathbf{P}}(k, t)=20 \\
\hline t
\end{array} r^{\operatorname{Margin}} \mathbf{P}(k, r)=30 \\
& k \quad k \quad \operatorname{Margin}_{\mathbf{P}}(r, k)=-30 \\
& r t \quad t \quad \operatorname{Margin}_{\mathbf{P}}(t, r)=-20 \\
& \operatorname{Margin}_{\mathbf{p}}(r, t)=20
\end{aligned}
$$



## Which candidate should be chosen?

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| :---: | :---: | :---: |
| $t$ | $r$ | $k$ |
| $k$ | $k$ | $r$ |
| $r$ | $t$ | $t$ |



Plurality winner $t$
Instant Runoff winner $r$
Condorcet winner $k$
Condorcet loser $t$

Suppose that $\mathbf{P}$ is an election (a record of the ballots submitted by the voters).

Suppose that $\mathbf{P}$ is an election (a record of the ballots submitted by the voters).
When more voters rank candidate $A$ above candidate $B$ than the other way around, the voters favor $A$ over $B$ in $\mathbf{P}$.

In most elections, the voting public favors one candidate over all others. This candidate is called the Condorcet winner.

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When more voters rank candidate $A$ above candidate $B$ than the other way around, the voters favor $A$ over $B$ in $\mathbf{P}$.

In most elections, the voting public favors one candidate over all others. This candidate is called the Condorcet winner.

- A candidate $a$ is the Condorcet winner in $\mathbf{P}$ when $a$ is majority preferred to every other candidate in $\mathbf{P}$.

Alternatively, For all candidates $b$ other than $a, \operatorname{Margin}_{\mathbf{P}}(a, b)>0$.
Alternatively, For all candidates $b$ other than $a$, there is an arrow from $a$ to $b$ in the majority (margin) graph for $\mathbf{P}$

