PHPE 400
Individual and Group Decision Making

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Nash Equilibria

- Some games may not have any pure strategy Nash equilibrium.
- Nash's Theorem: In any finite game, there is a mixed strategy Nash equilibrium.
- There may be more than one Nash equilibria.
- Components of Nash equilibria are not interchangeable: If $\mathbf{s}$ and $\mathbf{t}$ are Nash equilibria in a 2-player game, then $(\mathbf{s}_1, \mathbf{t}_2)$ may not be a Nash equilibrium.

Why *should* the players play their component of a Nash equilibrium?

When there are multiple Nash equilibria, how do the players decided which Nash equilibrium to play?

Why play Nash equilibrium?

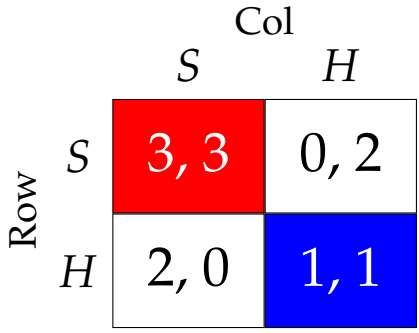
Self-Enforcing Agreements: Nash equilibria are recommended by being the only strategy combinations on which the players could make self-enforcing agreements, i.e., agreements that each has reason to respect, even without external enforcement mechanisms.

M. Risse. *What is rational about Nash equilibria?*. Synthese, 124:3, pgs. 361 - 384, 2000.

Stag-Hunt

Row \ Col	S	H
S	3, 3	0, 2
H	2, 0	1, 1

Stag-Hunt



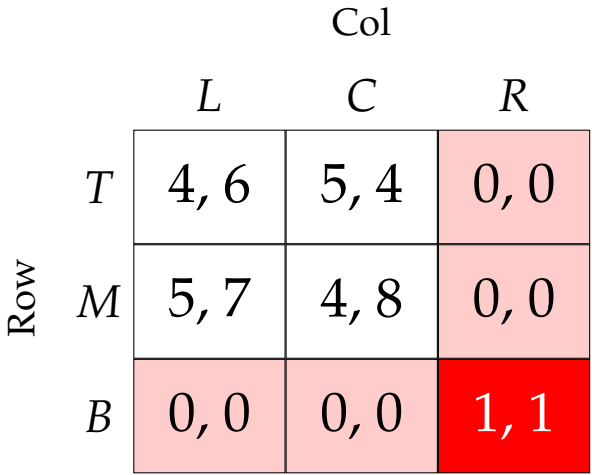
(S, S) and (H, H) are Nash equilibria

Stag-Hunt

Row \ Col	S	H
S	3, 3	0, 2
H	2, 0	1, 1

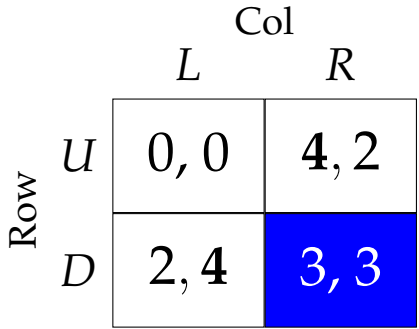
(S, S) is Pareto-superior, but (H, H) is less risky

Row \ Col	L	C	R
T	4, 6	5, 4	0, 0
M	5, 7	4, 8	0, 0
B	0, 0	0, 0	1, 1



(B, R) is a Nash equilibrium, but it is **not self-enforcing**

Row \ Col	L	R
U	0, 0	4, 2
D	2, 4	3, 3



(D,R) is self-enforcing, but **not a Nash equilibrium**

Self-Enforcing Agreements: Nash equilibria are recommended by being the only strategy combinations on which the players could make self-enforcing agreements, i.e., agreements that each has reason to respect, even without external enforcement mechanisms.

- Not all Nash equilibria are "equally" self-enforcing
- There are Nash equilibria that are not self-enforcing
- There are self-enforcing outcomes that are not Nash equilibria

Is a Nash equilibrium *guaranteed* by players that are rational rationality and have *common knowledge* of each others' rationality?

- Strategies that are not a Nash equilibrium may be *rationalizable*
- Sometimes considerations of riskiness trump the Nash equilibrium

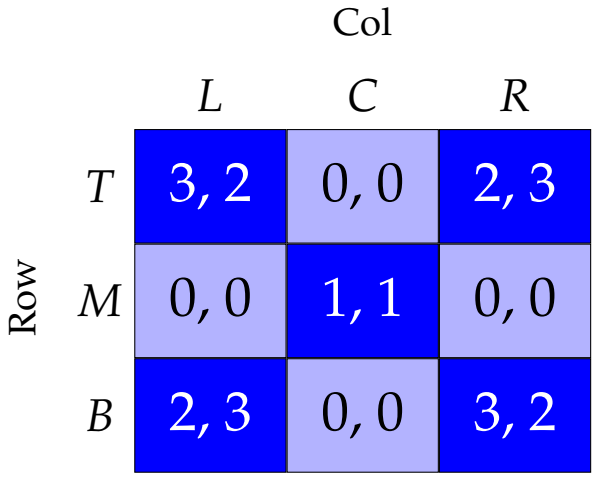
Row \ Col	L	C	R
T	3, 2	0, 0	2, 3
M	0, 0	1, 1	0, 0
B	2, 3	0, 0	3, 2

Row \ Col	L	C	R
T	3, 2	0, 0	2, 3
M	0, 0	1, 1	0, 0
B	2, 3	0, 0	3, 2

(M, C) is the unique Nash equilibrium

Row \ Col	L	C	R
T	3, 2	0, 0	2, 3
M	0, 0	1, 1	0, 0
B	2, 3	0, 0	3, 2

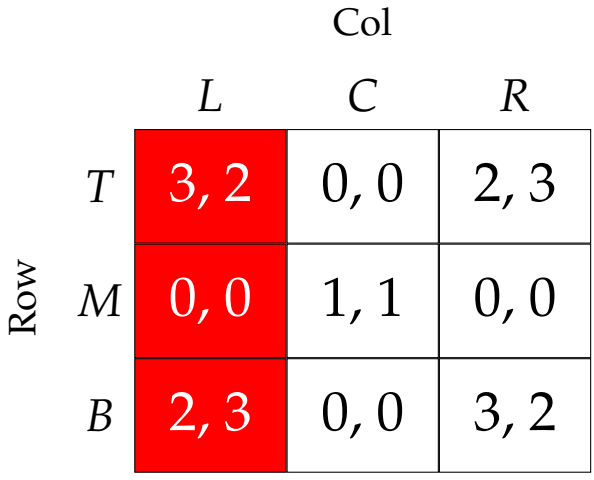
T, L, B and R are **rationalizable**



T, L, B and R are **rationalizable**

Row \ Col	*L*	*C*	*R*
T	3, 2	0, 0	2, 3
M	0, 0	1, 1	0, 0
B	2, 3	0, 0	3, 2

Row plays *B* because she thought Col will play *R*



Col plays L because she thought Row will play B

Row \ Col	*L*	*C*	*R*
T	3, 2	0, 0	2, 3
M	0, 0	1, 1	0, 0
B	2, 3	0, 0	3, 2

Col was correct, but Row was wrong

Row \ Col	*L*	*C*	*R*	*X*
T	3, 2	0, 0	2, 3	0, -5
M	0, 0	1, 1	0, 0	200,-5
B	2, 3	0, 0	3, 2	1,-3

Not every strategy is rationalizable: Row can't play *M because* she thinks Col will play *X*

An action A **strictly dominates** another action B for player i when i's utility is strictly better choosing A than choosing B no matter what actions are chosen by the other players.

Row \ Col	L	R
U	$5, \mathbf{5}$	$-100, 4$
D	$0, \mathbf{1}$	$0, 0$

Since R is **strictly dominated** by L, Column will not play R. Then, the best response for Row is U.