PHPE 400
Individual and Group Decision Making

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Strategic Games: Example

		Column	
		L	R
Row	U	2,1	0,0
	D	0,0	1,2

- $N = \{Row, Column\}$
- $A_{Row} = \{U, D\}$, $A_{Column} = \{L, R\}$
- $u_{Row} : A_{Row} \times A_{Column} \to \{0, 1, 2\}$, $u_{Column} : A_{Row} \times A_{Column} \to \{0, 1, 2\}$ with $u_{Row}(U, L) = u_{Column}(D, R) = 2$, $u_{Row}(D, R) = u_{Column}(U, L) = 1$, and $u_{Row}(D, L) = u_{Column}(D, L) = u_{Row}(U, R) = u_{Column}(U, R) = 0$.

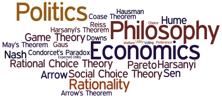
Strategy Profile

		Column	
		L	R
Row	U	2,1	0,0
	D	0,0	1,2

A **strategy profile** is a list of strategies, one for each player, that represents the outcome of the game.

The 4 possible strategy profiles in the above game are
$\{(U, L), (D, L), (U, R), (D, R)\}$

Pareto



A strategy profile **s Pareto dominates** a strategy profile **t** provided *every* player strictly prefers the outcome given **s** than the outcome given **t**.

For example, when there are two players, a strategy profile (A, B) **Pareto dominates** another strategy profile (X, Y) when

$$u_1(A, B) > u_1(X, Y) \text{ and } u_2(A, B) > u_2(X, Y).$$

A strategy profile **s** is **Pareto optimal** if **s** is not Pareto dominated by any other strategy profile.

Pareto

		Bob	
		L	R
Ann	U	1,1	0,0
	D	0,0	1,1

The strategy profile (U, L) Pareto dominates both (D, L) and (U, R).

But (U, L) does not Pareto dominate (D, R).

(U, L) and (D, R) are the Pareto optimal outcomes.

Pareto

Ann \ Bob	L	R
U	4,4	0,3
D	3,0	1,1

The strategy profile (U, L) Pareto dominates both (D, L) and (U, R).

But (U, L) Pareto dominates (D, R).

(U, L) is the unique Pareto optimal outcome.

Pareto

Ann \ Bob	L	R
U	1,-1	-1,1
D	-1,1	1,-1

All strategy profiles are Pareto optimal.

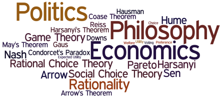
Solution Concept

A **solution concept** is a systematic description of the outcomes (i.e., the strategy profiles) that may emerge in a family of games.

This is the starting point for most of game theory and includes many variants: Nash equilibrium, backwards induction, or iterated dominance of various kinds.

These are usually thought of as the embodiment of “rational behavior” in some way and used to analyze game situations.

Best Response



For a player i and the list of strategies for the opponents, i's **best response** to these strategies is the strategy that maximizes i's utility *assuming the other players follow their given strategy*.

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	L	R
U	2,1	0,0
D	0,0	1,2

Row: The best response to L is U and the best response to R is D

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	L	R
U	2,1	0,0
D	0,0	1,2

Row: The best response to L is U and the best response to R is D

Column: The best response to U is L and the best response to D is R

Nash Equilibrium

A strategy profile is a **Nash equilibrium** if every player's strategy is a best response to the other player's strategies.

Nash Equilibrium: Example

	L	R
U	2,1	0,0
D	0,0	1,2

Row: The best response to L is U and the best response to R is D

Column: The best response to U is L and the best response to D is R

Nash Equilibrium: Example

	L	R
U	2,1	0,0
D	0,0	1,2

Row: The best response to L is U and the best response to R is D

Column: The best response to U is L and the best response to D is R

(U, L) is a Nash Equilibrium

(D, R) is a Nash Equilibrium

Matching Pennies

		Col H	Col T
Row	H	1,-1	-1, 1
Row	T	-1,1	1,-1

What are the players' best responses?

Matching Pennies

Row \ Col	H	T
H	1,-1	-1, 1
T	-1,1	1,-1

There are no pure strategy equilibrium.

Matching Pennies

Row \ Col	H	T
H	1,-1	-1, 1
T	-1,1	1,-1

Mixed Strategies

		Col	
		H	T
Row	H	1,-1	-1, 1
	T	-1,1	1,-1

A **mixed strategy** is a probability distribution over the set of pure strategies. For instance:

- $[H : 1/2, T : 1/2]$
- $[H : 1/3, T : 2/3]$
- ...

Matching Pennies

		Col H	Col T
Row	H	1,-1	-1, 1
Row	T	-1,1	1,-1

Consider the mixed strategy $([H : 1/2, T : 1/2], [H : 1/2, T : 1/2])$.

Matching Pennies

		Col	
		H	*T*
Row	*H*	1,-1	-1, 1
	T	-1,1	1,-1

The mixed strategy $([H : 1/2, T : 1/2], [H : 1/2, T : 1/2])$ is the only *mixed-strategy* Nash equilibrium.

Row \ Col	L	R
U	2, 1	0, 0
D	0, 0	1, 2

(U, L), (D, R), and $([U : 2/3, D : 1/3], [L : 1/3, R : 2/3])$ are Nash equilibria.

Mixed Strategies

"We are reluctant to believe that our decisions are made at random. We prefer to be able to point to a reason for each action we take. Outside of Las Vegas we do not spin roulettes."

A. Rubinstein. *Comments on the Interpretation of Game Theory*. Econometrica 59, 909 - 924, 1991.

What does it mean to play a mixed strategy? Different interpretations:

- Randomize to confuse your opponent (e.g., matching pennies games)
- Players randomize when they are uncertain about the other's action (e.g., battle of the sexes game)
- Mixed strategies are a concise description of what might happen in repeated play
- Mixed strategies describe population dynamics: After selecting 2 agents from a population, a mixed strategy is the probability of getting an agent who will play one pure strategy or another.

Nash Equilibria

- Some games may not have any pure strategy Nash equilibrium.
- Nash's Theorem: In any finite game, there is a mixed strategy Nash equilibrium.
- There may be more than one Nash equilibria.
- Components of Nash equilibria are not interchangeable: If $\mathbf{s}$ and $\mathbf{t}$ are Nash equilibria in a 2-player game, then $(\mathbf{s}_1, \mathbf{t}_2)$ may not be a Nash equilibrium.

Why *should* the players play their component of a Nash equilibrium?

When there are multiple Nash equilibria, how do the players decided which Nash equilibrium to play?

Why play Nash equilibrium?

Self-Enforcing Agreements: Nash equilibria are recommended by being the only strategy combinations on which the players could make self-enforcing agreements, i.e., agreements that each has reason to respect, even without external enforcement mechanisms.

M. Risse. *What is rational about Nash equilibria?*. Synthese, 124:3, pgs. 361 - 384, 2000.