# PHPE 400 <br> Individual and Group Decision Making 

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## Game Theory

## The Guessing Game

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Guess a number between $1 \& 100$. The closest to $2 / 3$ of the average wins.

## Traveler's Dilemma

1. You and your friend write down an integer between 2 and 100 (without discussing).
2. If both of you write down the same number, then both will receive that amount in dollars from the airline in compensation.
3. If the numbers are different, then the airline assumes that the smaller number is the actual price of the luggage.
4. The person that wrote the smaller number will receive that amount plus $\$ 2$ (as a reward), and the person that wrote the larger number will receive the smaller number minus $\$ 2$ (as a punishment).

Suppose that you are randomly paired with another person from class. What number would you write down?

## From Decisions to Games

What makes the previous decision problems different from standard decision problems?

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"[T]he fundamental insight of game theory [is] that a rational player must take into account that the players reason about each other in deciding how to play."
R. Aumann and J. Dreze. Rational Expectations in Games. American Economic Review, 98, pp. 72-86, 2008.


Red wine
White wine
Steak
Fish






## Game Situations

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a group of self-interested agents (players) involved in some interdependent decision problem

## Game Situations

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## $\begin{array}{lll}\text { E } & F & 2 \\ \\ I & 0 & 0 \\ & 1\end{array}$

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## Game Situations

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## Bob <br> $F \quad I$ <br> 

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## Game Situations

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## Game Situations


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## Just Enough Game Theory

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## Just Enough Game Theory

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Theory Arrow Rationality

A game is a mathematical model of a strategic interaction that includes

- the group of players in the game


## Just Enough Game Theory

A game is a mathematical model of a strategic interaction that includes

- the group of players in the game
- the actions the players can take


## Just Enough Game Theory

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- the group of players in the game
- the actions the players can take
- the players' interests (i.e., preferences/utilities),


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- the "structure" of the decision problem (what information do the players have?, what order do they act in?, how many times do they repeat their interaction?, etc.)


## Just Enough Game Theory

A game is a mathematical model of a strategic interaction that includes

- the group of players in the game
- the actions the players can take
- the players' interests (i.e., preferences/utilities),
- the "structure" of the decision problem (what information do the players have?, what order do they act in?, how many times do they repeat their interaction?, etc.)

It does not specify the actions that the players do take.

## Simultaneous-move

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- In simultaneous-move games all players select an action at the same time, without knowing what the others will do (though they can certainly reason about what the other players should be expected to do).


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## Strategic Games

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A strategic game is a tuple $\left\langle N,\left(A_{i}\right)_{i \in N},\left(u_{i}\right)_{i \in N}\right\rangle$ where

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## Strategic Games

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- $N$ is a finite set of players
- for each $i \in N, A_{i}$ is a nonempty set of actions
- for each $i \in N, u_{i}$ is a utility function for player $i$ on $A=\Pi_{i \in N} A_{i}$ $u_{i}: A \rightarrow \mathbb{R}$


## Strategic Games: Example

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- $N=\{$ Row, Column $\}$
- $A_{\text {Row }}=\{U, D\}, A_{\text {column }}=\{L, R\}$
- $u_{\text {Row }}: A_{\text {Row }} \times A_{\text {Column }} \rightarrow\{0,1,2\}, u_{\text {Column }}: A_{\text {Row }} \times A_{\text {Column }} \rightarrow\{0,1,2\}$ with $u_{\text {Row }}(U, L)=u_{\text {Column }}(D, R)=2, u_{\text {Row }}(D, R)=u_{\text {Column }}(U, L)=1$, and $u_{\text {Row }}(D, L)=u_{\text {Column }}(D, L)=u_{\text {Row }}(U, R)=u_{\text {Column }}(U, R)=0$.


## Strategy Profile

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## Column <br> 

A strategy profile is a list of strategies, one for each player, that represents the outcome of the game.

The 4 possible strategy profiles in the above game are $\{(U, L),(D, L),(U, R),(D, R)\}$

## Pareto

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A strategy profile s Pareto dominates a strategy profile $\boldsymbol{t}$ provided every player strictly prefers the outcome given $\mathbf{s}$ than the outcome given $\mathbf{t}$.

For example, when there are two players, a strategy profile $(A, B)$ Pareto dominates another strategy profile $(X, Y)$ when

$$
u_{1}(A, B)>u_{1}(X, Y) \text { and } u_{2}(A, B)>u_{2}(X, Y) .
$$

A strategy profile $\mathbf{s}$ is Pareto optimal if $\mathbf{s}$ is not Pareto dominated by any other strategy profile.

## Pareto

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## Bob <br> 

The strategy profile $(U, L)$ Pareto dominates both $(D, L)$ and $(U, R)$. But $(U, L)$ does not Pareto dominate $(D, R)$.
$(U, L)$ and $(D, R)$ are the Pareto optimal outcomes.

## Pareto

 Mass Game theory Arrow Rationality


The strategy profile $(U, L)$ Pareto dominates both $(D, L)$ and $(U, R)$. But $(U, L)$ Pareto dominates $(D, R)$.
$(U, L)$ is the unique Pareto optimal outcome.

## Pareto

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## Bob <br> L $\quad$ R <br> $$
\mathbb{E}^{U} \begin{array}{l|l|l|} \hline & 1,-1 & -1,1 \\ D & -1,1 & 1,-1 \\ \hline \end{array}
$$

All strategy profiles are Pareto optimal.

## Solution Concept

A solution concept is a systematic description of the outcomes (i.e., the strategy profiles) that may emerge in a family of games.

This is the starting point for most of game theory and includes many variants: Nash equilibrium, backwards induction, or iterated dominance of various kinds.

These are usually thought of as the embodiment of "rational behavior" in some way and used to analyze game situations.

## Best Response

For a player $i$ and a strategy $s$ of the opponents, $B R_{i}(s)$ is $i$ 's best response to $s$ : The strategy that maximizes $i$ 's utility assuming the other players follow strategys.

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N=\{r, c\} \quad A_{r}=\{U, D\} \quad A_{c}=\{L, R\}
$$

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|  | L | $R$ |
| :---: | :---: | :---: |
| $U$ | 2,1 | 0,0 |
| D | 0,0 | 1,2 |

$$
\begin{array}{cc}
N=\{r, c\} \quad A_{r}=\{U, D\} & A_{c}=\{L, R\} \\
B R_{r}(L)=\{U\} & B R_{r}(R)=\{D\}
\end{array}
$$

## Best Response

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|  | $L$ | $R$ |
| :---: | :---: | :---: |
| $U$ | 2,1 | 0,0 |
| $D$ | 0,0 | 1,2 |
|  |  |  |

$$
\begin{array}{cc}
N=\{r, c\} \quad A_{r}=\{U, D\} & A_{c}=\{L, R\} \\
B R_{r}(L)=\{U\} & B R_{r}(R)=\{D\} \\
B R_{c}(U)=\{L\} & B R_{c}(D)=\{R\}
\end{array}
$$

## Nash Equilibrium

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A strategy profile is a Nash equilibrium if every player's strategy is a best response to the other player's strategies.

## Example

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## Example

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\begin{array}{cc}
N=\{r, c\} \quad A_{r}=\{U, D\} & A_{c}=\{L, R\} \\
B R_{r}(L)=\{U\} & B R_{r}(R)=\{D\}
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## Example

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$$
\begin{array}{cc}
N=\{r, c\} \quad A_{r}=\{U, D\} & A_{c}=\{L, R\} \\
B R_{r}(L)=\{U\} & B R_{r}(R)=\{D\} \\
B R_{c}(U)=\{L\} & B R_{c}(D)=\{R\}
\end{array}
$$

## Example

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$(U, L)$ is a Nash Equilibrium
$(D, R)$ is a Nash Equilibrium

$(U, L)$ and $(D, R)$ are Nash equilibria.

## Matching Pennies

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What are the players' best responses?

## Matching Pennies

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There are no pure strategy equilibrium.

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## Mixed Strategies

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$$
\begin{aligned}
& \mathrm{Col} \\
& \text { H T } \\
& \begin{array}{|c|c|c|}
{ }^{2} \\
\cline { 2 - 3 } \\
\cline { 2 - 3 } & 1,-1 & -1,1 \\
& -1,1 & 1,-1 \\
\hline
\end{array}
\end{aligned}
$$

A mixed strategy is a probability distribution over the set of pure strategies. For instance:

- $[H: 1 / 2, T: 1 / 2]$
- $[H: 1 / 3, T: 2 / 3]$


## Matching Pennies

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Consider the mixed strategy $([H: 1 / 2, T: 1 / 2],[H: 1 / 2, T: 1 / 2])$.

## Matching Pennies


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Rationality

The mixed strategy $([H: 1 / 2, T: 1 / 2],[H: 1 / 2, T: 1 / 2])$ is the only mixed-strategy Nash equilibrium.


[^0]:    pictured above: Bach/Stravinsky Game (also called Battle of the Sexes)

