PHPE 400
Individual and Group Decision Making

Eric Pacuit
University of Maryland
pacuit.org

Game Theory

The Guessing Game

Guess a number between 1 & 100.
The closest to 2/3 of the average wins.

Traveler's Dilemma

1. You and your friend write down an integer between 2 and 100 (without discussing).
2. If both of you write down the same number, then both will receive that amount in dollars from the airline in compensation.
3. If the numbers are different, then the airline assumes that the smaller number is the actual price of the luggage.
4. The person that wrote the smaller number will receive that amount plus $2 (as a reward), and the person that wrote the larger number will receive the smaller number minus $2 (as a punishment).

Suppose that you are randomly paired with another person from class. What number would you write down?

From Decisions to Games

What makes the previous decision problems different from standard decision problems?

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"*[T]he* fundamental insight of game theory [is] that a rational player must take into account that the players reason about each other in deciding how to play."

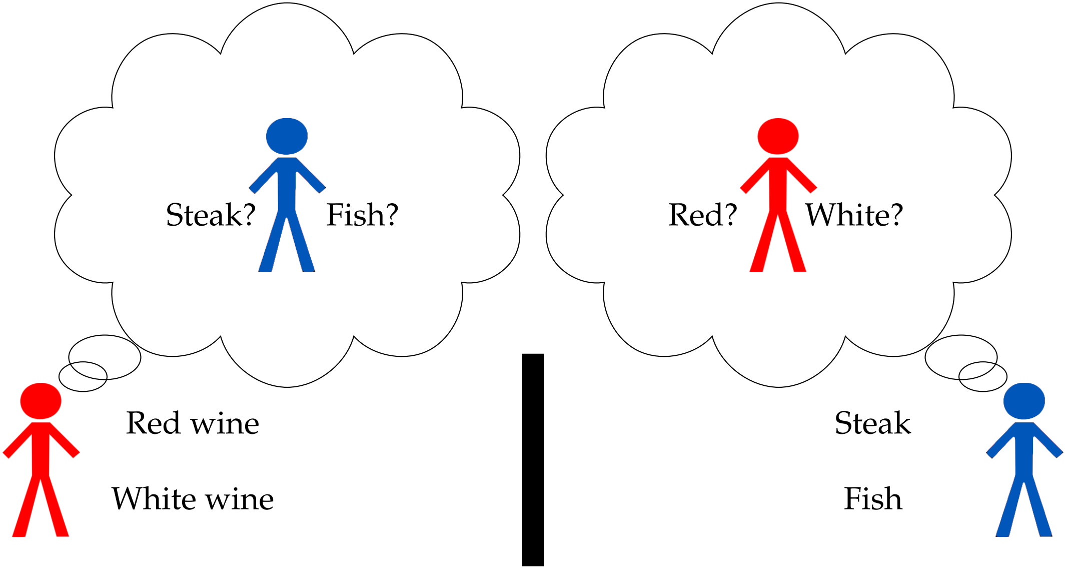
R. Aumann and J. Dreze. *Rational Expectations in Games*. American Economic Review, 98, pp. 72-86, 2008.

Red wine

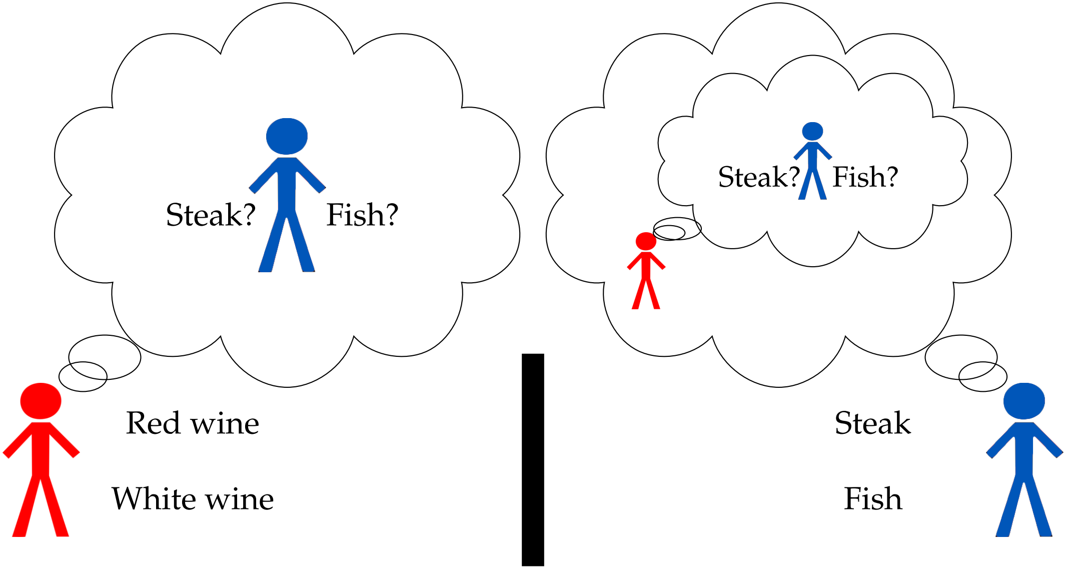
White wine

Steak

Fish



Steak?
Fish?
Red?
White?
Red wine
White wine
Steak
Fish


Steak?
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Red wine
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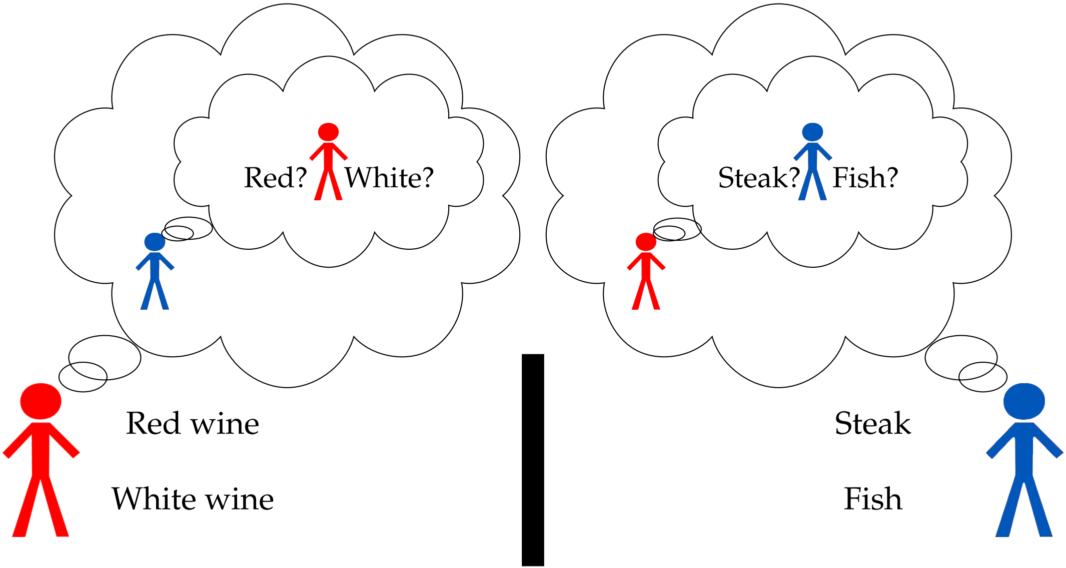
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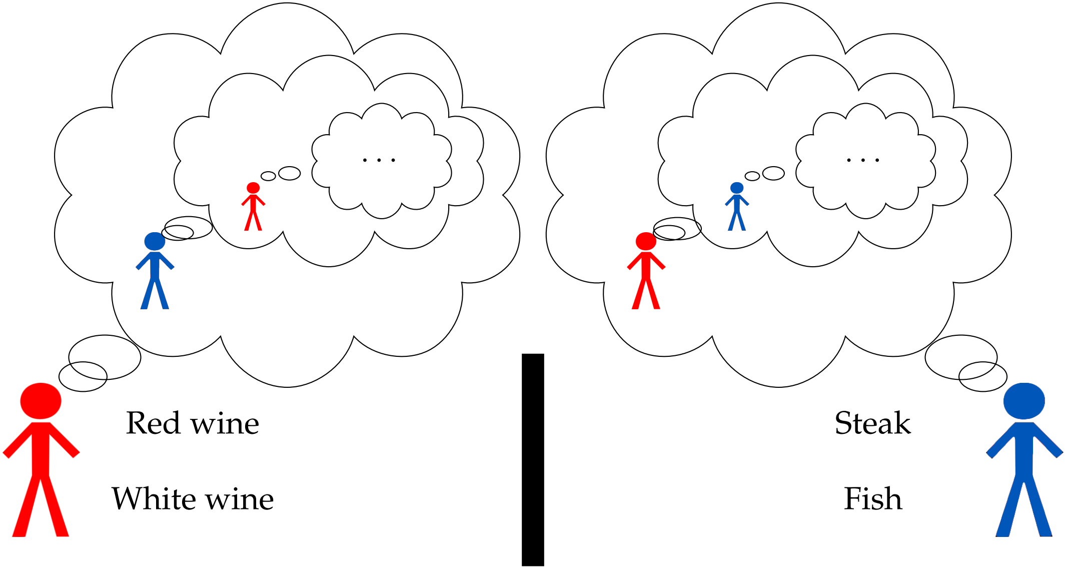

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Red wine
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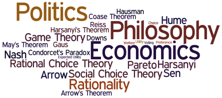
Game Situations

a group of *self-interested* agents (players) involved in some interdependent decision problem

Game Situations



Ann	F	2	0
	I	0	1

a group of *self-interested* agents (players) involved in some interdependent decision problem

Game Situations

		Bob: *F*	Bob: *I*
Ann	*F*	2 1	0 0
Ann	*I*	0 0	1 2

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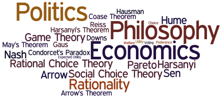
Game Situations



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pictured above: Bach/Stravinsky Game (also called Battle of the Sexes)

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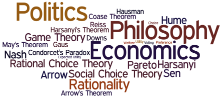

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- the group of players in the game
- the actions the players *can* take
- the players' interests (i.e., preferences/utilities),
- the "structure" of the decision problem (what information do the players have?, what order do they act in?, how many times do they repeat their interaction?, etc.)

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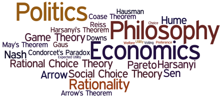

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- the actions the players *can* take
- the players' interests (i.e., preferences/utilities),
- the "structure" of the decision problem (what information do the players have?, what order do they act in?, how many times do they repeat their interaction?, etc.)

*It does **not** specify the actions that the players **do take**.*

Simultaneous-move

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Strategic Games

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- N is a finite set of **players**
- for each $i \in N$, A_i is a nonempty set of **actions**
- for each $i \in N$, u_i is a **utility function** for player i on $A = \Pi_{i \in N} A_i$
  $u_i : A \to \mathbb{R}$

Strategic Games: Example

		Column L	Column R
Row	U	2,1	0,0
Row	D	0,0	1,2

- $N = \{Row, Column\}$
- $A_{Row} = \{U, D\}$, $A_{Column} = \{L, R\}$
- $u_{Row} : A_{Row} \times A_{Column} \to \{0, 1, 2\}$, $u_{Column} : A_{Row} \times A_{Column} \to \{0, 1, 2\}$ with $u_{Row}(U, L) = u_{Column}(D, R) = 2$, $u_{Row}(D, R) = u_{Column}(U, L) = 1$, and $u_{Row}(D, L) = u_{Column}(D, L) = u_{Row}(U, R) = u_{Column}(U, R) = 0$.

Strategy Profile

Column

Row		L	R
	U	2,1	0,0
	D	0,0	1,2

A **strategy profile** is a list of strategies, one for each player, that represents the outcome of the game.

The 4 possible strategy profiles in the above game are
$\{(U, L), (D, L), (U, R), (D, R)\}$

Pareto

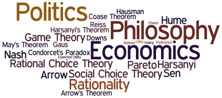
A strategy profile **s** **Pareto dominates** a strategy profile **t** provided *every* player strictly prefers the outcome given **s** than the outcome given **t**.

For example, when there are two players, a strategy profile (A, B) **Pareto dominates** another strategy profile (X, Y) when

$$u_1(A, B) > u_1(X, Y) \text{ and } u_2(A, B) > u_2(X, Y).$$

A strategy profile **s** is **Pareto optimal** if **s** is not Pareto dominated by any other strategy profile.

Pareto

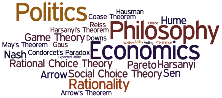


Ann \ Bob	L	R
U	1,1	0,0
D	0,0	1,1

The strategy profile (U, L) Pareto dominates both (D, L) and (U, R).

But (U, L) does not Pareto dominate (D, R).

(U, L) and (D, R) are the Pareto optimal outcomes.



Pareto

		Bob	
		L	R
Ann	U	4,4	0,3
	D	3,0	1,1

The strategy profile (U, L) Pareto dominates both (D, L) and (U, R).

But (U, L) Pareto dominates (D, R).

(U, L) is the unique Pareto optimal outcome.

Pareto

		Bob	
		L	*R*
Ann	*U*	1,-1	-1,1
	D	-1,1	1,-1

All strategy profiles are Pareto optimal.

Solution Concept

A **solution concept** is a systematic description of the outcomes (i.e., the strategy profiles) that may emerge in a family of games.

This is the starting point for most of game theory and includes many variants: Nash equilibrium, backwards induction, or iterated dominance of various kinds.

These are usually thought of as the embodiment of “rational behavior” in some way and used to analyze game situations.

Best Response

For a player i and a strategy s of the opponents, $BR_i(s)$ is i's **best response** to s: The strategy that maximizes i's utility *assuming the other players follow strategy* s.

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$$BR_c(U) = \{L\} \qquad BR_c(D) = \{R\}$$

Nash Equilibrium

A strategy profile is a **Nash equilibrium** if every player’s strategy is a best response to the other player’s strategies.

Example

	L	R
U	2,1	0,0
D	0,0	1,2

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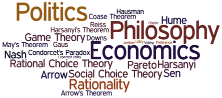

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$$BR_r(L) = \{U\} \qquad BR_r(R) = \{D\}$$

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(U, L) is a Nash Equilibrium $\qquad$ (D, R) is a Nash Equilibrium

		Bob	
		L	R
Ann	U	4,4	0,3
	D	3,0	1,1

(U, L) and (D, R) are Nash equilibria.

Matching Pennies

Row \ Col	H	T
H	1,-1	-1, 1
T	-1,1	1,-1

What are the players' best responses?

Matching Pennies

		Col	
		H	T
Row	H	1,-1	-1, 1
	T	-1,1	1,-1

There are no pure strategy equilibrium.

Matching Pennies

		Col *H*	Col *T*
Row	*H*	1,-1	-1, 1
Row	*T*	-1,1	1,-1

Mixed Strategies

Row \ Col	H	T
H	1,-1	-1, 1
T	-1,1	1,-1

A **mixed strategy** is a probability distribution over the set of pure strategies. For instance:

- $[H : 1/2, T : 1/2]$
- $[H : 1/3, T : 2/3]$
- ...

Matching Pennies

Row \ Col	H	T
H	1,-1	-1, 1
T	-1,1	1,-1

Consider the mixed strategy $([H : 1/2, T : 1/2], [H : 1/2, T : 1/2])$.

Matching Pennies

		Col	
		H	T
Row	H	1,-1	-1, 1
	T	-1,1	1,-1

The mixed strategy $([H : 1/2, T : 1/2], [H : 1/2, T : 1/2])$ is the only *mixed-strategy* Nash equilibrium.