PHPE 400
Individual and Group Decision Making

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Allais Paradox

		Red (1)	White (89)	Blue (10)
S_1	A	$1M$	$1M$	$1M$
	B	0	$1M$	$5M$

Allais Paradox

		Red (1)	White (89)	Blue (10)
S_2	C	$1M$	0	$1M$
	D	0	0	$5M$

Allais Paradox

		Red (1)	White (89)	Blue (10)
S_1	A	$1M$	$1M$	$1M$
	B	0	$1M$	$5M$
S_2	C	$1M$	0	$1M$
	D	0	0	$5M$

Independence and Allais

$$[1M : \tfrac{1}{100}, 1M : \tfrac{89}{100}, 1M : \tfrac{10}{100}] \quad P \quad [0 : \tfrac{1}{100}, 1M : \tfrac{89}{100}, 5M : \tfrac{10}{100}]$$

Independence and Allais

$$[1M : \tfrac{1}{100}, 1M : \tfrac{89}{100}, 1M : \tfrac{10}{100}] \quad P \quad [0 : \tfrac{1}{100}, 1M : \tfrac{89}{100}, 5M : \tfrac{10}{100}]$$

iff

$$[[1M : \tfrac{1}{11}, 1M : \tfrac{10}{11}] : \tfrac{11}{100}, [1M : 1] : \tfrac{89}{100}] \quad P \quad [[0 : \tfrac{1}{11}, 5M : \tfrac{10}{11}] : \tfrac{11}{100}, [1M : 1] : \tfrac{89}{100}]$$

Independence and Allais

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Independence and Allais

$$[1M:\tfrac{1}{100},1M:\tfrac{89}{100},1M:\tfrac{10}{100}] \quad P \quad [0:\tfrac{1}{100},1M:\tfrac{89}{100},5M:\tfrac{10}{100}]$$

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Independence and Allais

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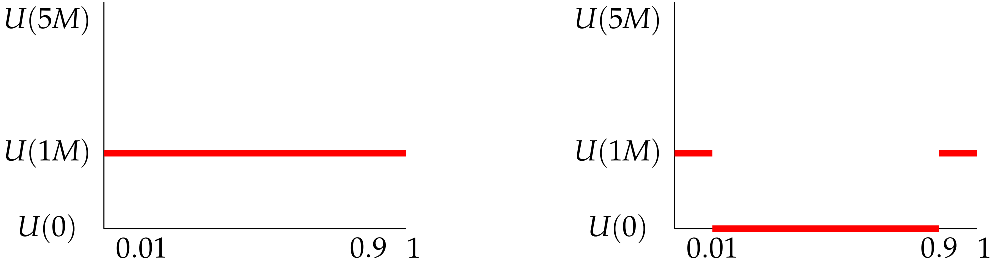
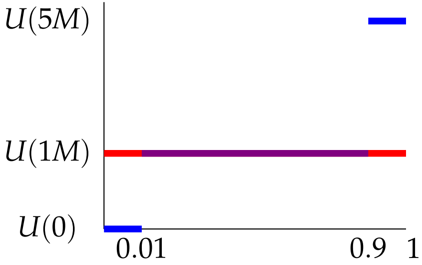
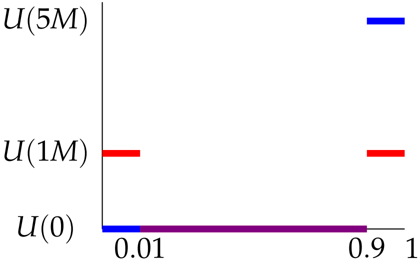


$[\ 1M : 0.01, \quad 1M : 0.89, \quad 1M : 0.1\ ]$ $[\ 1M : 0.01, \quad 0 : 0.89, \quad 1M : 0.1\ ]$





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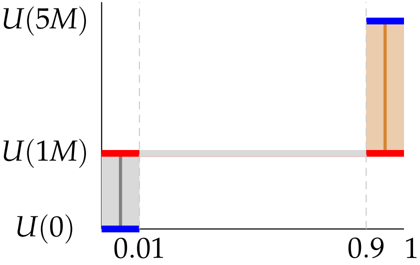

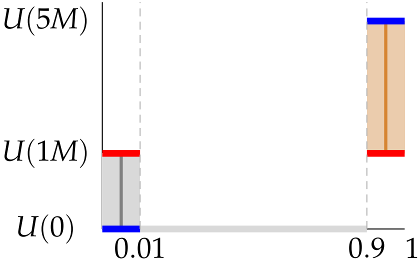

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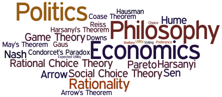
	B	0	$1M$	$5M$
S_2	C	$1M$	0	$1M$
	D	0	0	$5M$

$A \; P \; B$ if and only if $C \; P \; D$

Allais Paradox

We should not conclude either

Allais Paradox



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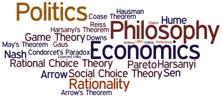
Allais Paradox



We should not conclude either

(a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or

(b) those who choose A in S_1 and D is S_2 are irrational.

Rather, people's utility functions (*their rankings over outcomes*) are often far more complicated than the monetary bets would indicate....

L. Buchak. *Risk and Rationality*. Oxford University Press, 2013.

Ellsberg Paradox

	30	60	
Lotteries	Blue	Yellow	Green
L_1	$1M$	0	0
L_2	0	$1M$	0

Ellsberg Paradox

	30	60	
Lotteries	Blue	Yellow	Green
L_3	$1M$	0	$1M$
L_4	0	$1M$	$1M$

Ellsberg Paradox

	30	60	
Lotteries	Blue	Yellow	Green
L_1	$1M$	0	0
L_2	0	$1M$	0
L_3	$1M$	0	$1M$
L_4	0	$1M$	$1M$

$L_1 \; R \; L_2$ if and only if $L_3 \; R \; L_4$

Let r be any integer between 30 and 60 (i.e., $30 \leq r \leq 60$) and $q = 90 - 30 - r$

$$[1M : \tfrac{30}{90}, 0 : \tfrac{r}{90}, 0 : \tfrac{q}{90}] \quad P \quad [0 : \tfrac{30}{90}, 1M : \tfrac{r}{90}, 0 : \tfrac{q}{90}]$$

iff

$$[[1M : \tfrac{30}{30+r}, 0 : \tfrac{r}{30+r}] : \tfrac{30+r}{90}, 0 : \tfrac{q}{90}] \quad P \quad [[0 : \tfrac{30}{30+r}, 1M : \tfrac{r}{30+r}] : \tfrac{30+r}{90}, 0 : \tfrac{q}{90}]$$

iff

$$[1M : \tfrac{30}{30+r}, 0 : \tfrac{r}{30+r}] \quad P \quad [0 : \tfrac{30}{30+r}, 1M : \tfrac{r}{30+r}]$$

iff

$$[[1M : \tfrac{30}{30+r}, 0 : \tfrac{r}{30+r}] : \tfrac{30+r}{90}, 1M : \tfrac{q}{90}] \quad P \quad [[0 : \tfrac{30}{30+r}, 1M : \tfrac{r}{30+r}] : \tfrac{30+r}{90}, 1M : \tfrac{q}{90}]$$

iff

$$[1M : \tfrac{30}{90}, 0 : \tfrac{r}{90}, 1M : \tfrac{q}{90}] \quad P \quad [0 : \tfrac{30}{90}, 1M : \tfrac{r}{90}, 1M : \tfrac{q}{90}]$$

Ambiguity Aversion

I. Gilboa and M. Marinacci. *Ambiguity and the Bayesian Paradigm*. Advances in Economics and Econometrics: Theory and Applications, Tenth World Congress of the Econometric Society. D. Acemoglu, M. Arellano, and E. Dekel (Eds.). New York: Cambridge University Press, 2013.

Flipping a fair coin vs. flipping a coin of unknown bias

Decision problems

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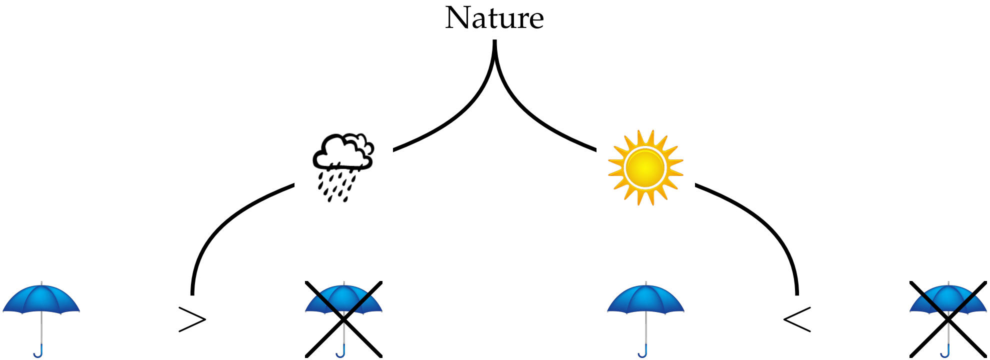
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	encumbered, dry	encumbered, dry
	wet	free, dry

States: it rains; it does not rain

Outcomes: encumbered, dry; wet; free, dry

Actions: take umbrella; leave umbrella

		
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States: it rains; it does not rain

Outcomes: encumbered, dry; wet; free, dry

Actions: take umbrella; leave umbrella

	Rain (s_1)	No rain (s_2)
Take umbrella (A)	encumbered, dry (o_1)	encumbered, dry (o_1)
Leave umbrella (B)	free, wet (o_2)	free, dry (o_3)

$$A(s_1) = A(s_2) = o_1$$

$$B(s_1) = o_2, B(s_2) = o_3$$

	Rain (s_1)	No rain (s_2)
Take umbrella (A)	encumbered, dry (o_1)	encumbered, dry (o_1)
Leave umbrella (B)	free, wet (o_2)	free, dry (o_3)

Suppose that $P(s_1) = 0.6$ and $P(s_2) = 0.4$
(the decision maker believes that there is a 60% chance that it will rain).

	Rain (s_1)	No rain (s_2)
Take umbrella (A)	encumbered, dry (o_1)	encumbered, dry (o_1)
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Suppose that $P(s_1) = 0.6$ and $P(s_2) = 0.4$
(the decision maker believes that there is a 60% chance that it will rain).

Suppose that the decision maker's utility for the outcomes is:
$u(o_1) = 5$, $u(o_2) = 0$ and $u(o_3) = 10$.

	Rain (s_1) $P(s_1) = 0.6$	No rain (s_2) $P(s_2) = 0.4$
Take umbrella (A)	encumbered, dry (o_1) $u(o_1) = 5$	encumbered, dry (o_1) $u(o_1) = 5$
Leave umbrella (B)	free, wet (o_2) $u(o_2) = 0$	free, dry (o_3) $u(o_3) = 10$

$$EU(A) = 0.6 * 5 + 0.4 * 5 = 5 > EU(B) = 0.6 * 0 + 0.4 * 10 = 4$$

	Rain (s_1) $P(s_1) = 0.6$	No rain (s_2) $P(s_2) = 0.4$
Take umbrella (A)	encumbered, dry (o_1) $u'(o_1) = 4$	encumbered, dry (o_1) $u'(o_1) = 4$
Leave umbrella (B)	free, wet (o_2) $u'(o_2) = 2$	free, dry (o_3) $u'(o_3) = 8$

$$EU(A) = 0.6 * 4 + 0.4 * 4 = 4 < EU(B) = 0.6 * 2 + 0.4 * 8 = 1.2 + 3.2 = 4.4$$

	Rain (s_1) $P(s_1) = 0.6$	No rain (s_2) $P(s_2) = 0.4$
Take umbrella (A)	encumbered, dry (o_1)	encumbered, dry (o_1)
Leave umbrella (B)	free, wet (o_2)	free, dry (o_3)

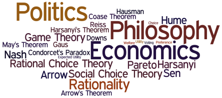
$$u(o_3) = 10 > u(o_1) = 5 > u(o_2) = 0$$
$$EU(A) = 0.6 * 5 + 0.4 * 5 = 5 > EU(B) = 0.6 * 0 + 0.4 * 10 = 4$$

$$u'(o_3) = 8 > u'(o_1) = 4 > u'(o_2) = 2$$
$$EU(A) = 0.6 * 4 + 0.4 * 4 = 4 < EU(B) = 0.6 * 2 + 0.4 * 8 = 1.2 + 3.2 = 4.4$$

For all acts A and B and utility functions u,

if $EU(A, u) > EU(B, u)$ and u' is a linear transformation of u (i.e., $u'(\cdot) = au(\cdot) + b$ for some $a, b \in \mathbb{R}$), then $EU(A, u') > EU(B, u')$

Strict Dominance



	s_1	s_2	s_3
A	2	3	1
B	1	2	0
C	1	4	0

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers B to A?

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers C to A?

Strict Dominance

	s_1	s_2	s_3
A	2	3	1
B	1	2	0
C	1	4	0

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers B to A? No!

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers C to A? Yes!

Strict Dominance

	s_1	s_2	s_3
A	2	3	1
B	1	2	0
C	1	4	0

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers B to A? No!

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers C to A? Yes!

X **strictly dominates** Y when for all states s, $u(X(s)) > u(Y(s))$.

- A strictly dominates B

Strict Dominance

	s_1	s_2	s_3
A	2	3	1
B	1	2	0
C	1	4	0

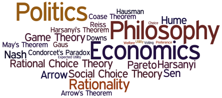
Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers B to A? No!

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers C to A? Yes!

X **strictly dominates** Y when for all states s, $u(X(s)) > u(Y(s))$.

- A strictly dominates B
- A does not strictly dominate C

Weak Dominance

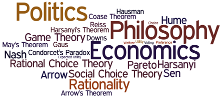


	s_1	s_2	s_3
A	2	3	1
B	1	2	1
C	2	3	1

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers B to A?

Is there a way of assigning probabilities to the states s_1, s_2, and s_3 such that the decision maker strictly prefers C to A?

Weak Dominance



	s_1	s_2	s_3
A	2	3	1
B	1	2	1
C	2	3	1

Does the decision maker strictly prefer A to B?

Does the decision maker strictly prefer A to C?

Weak Dominance

	s_1	s_2	s_3
A	2	3	1
B	1	2	1
C	2	3	1

Does the decision maker strictly prefer A to B? Depends...

Does the decision maker strictly prefer A to C? No!

X **weakly dominates** Y when for all states s, $u(X(s)) \geq u(Y(s))$ and there is some s' such that $u(X(s')) > u(Y(s'))$.

- A weakly dominates B

Weak Dominance

	s_1	s_2	s_3
A	2	3	1
B	1	2	1
C	2	3	1

Does the decision maker strictly prefer A to B? Depends...

Does the decision maker strictly prefer A to C? No!

X **weakly dominates** Y when for all states s, $u(X(s)) \geq u(Y(s))$ and there is some s' such that $u(X(s')) > u(Y(s'))$.

- A weakly dominates B
- A does not weakly dominate C