# PHPE 400 <br> Individual and Group Decision Making 

Eric Pacuit<br>University of Maryland<br>pacuit.org

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A rational preference over lotteries involves more than the assumption that the decision maker's preferences are transitive and complete:

1. Independence axiom
2. Compound lottery axiom
3. Continuity axiom

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Suppose that the decision maker is rational and has the preference $a P b$ (the decision maker strictly prefers $a$ to $b$ ).

How should the decision maker rank the lotteries
$L_{1}=[a: 0.6, b: 0.4]$ and $L_{2}=[a: 0.4, b: 0.6]$ ?

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1. $L_{1} P L_{2}$ : The decision maker should strictly prefer $L_{1}$ to $L_{2}$.
2. $L_{2} P L_{1}$ : The decision maker should strictly prefer $L_{2}$ to $L_{1}$.
3. $L_{1} I L_{2}$ : The decision maker should be indifferent between $L_{1}$ and $L_{2}$.
4. There is not enough information to answer this question.

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Suppose that the decision maker is rational and has the preference $a P b$ (the decision maker strictly prefers $a$ to $b$ ) and $c$ is another item.

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Suppose that the decision maker is rational and has the preference $a P b$ (the decision maker strictly prefers $a$ to $b$ ) and $c$ is another item.

Then, a rational decision maker will have the following preferences:

1. The decision maker strictly prefers $[a: 0.6, b: 0.4]$ over $[a: 0.4, b: 0.6]$
2. The decision maker strictly prefers $[a: 0.6, c: 0.4]$ over $[a: 0.6, c: 0.4]$

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1. The decision maker strictly prefers $[a: 0.6, b: 0.4]$ over $[a: 0.4, b: 0.6]$
2. The decision maker strictly prefers $[a: 0.6, c: 0.4]$ over $[a: 0.6, c: 0.4]$

Neither of these preferences can be inferred if all you know is that the decision maker's preferences over lotteries satisfies transitivity and completeness.

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## Independence

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For all $L_{1}, L_{2}, L_{3} \in \mathcal{L}$ and $0<p \leq 1$,

$$
L_{1} P L_{2} \text { if, and only if, }\left[L_{1}: p, L_{3}:(1-p)\right] P\left[L_{2}: p, L_{3}:(1-p)\right] .
$$

$L_{1} I L_{2}$ if, and only if, $\left[L_{1}: p, L_{3}:(1-p)\right] I\left[L_{2}: p, L_{3}:(1-p)\right]$.

## Independence



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For all $\square, \square \in \mathcal{L}$ and $0<p \leq 1$, $\square P \square$ if, and only if, $\square: p, \square:(1-p)] P[\square: p, \square:(1-p)]$.

$$
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$$

## Combining Lotteries

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Suppose that $L_{1}=[a: 0.5, b: 0.5]$ and $L_{2}=[b: 0.25, c: 0.75]$
There are many ways to combine these lotteries.
E.g., $\left[L_{1}: 0.2, L_{2}: 0.8\right]=[[a: 0.5, b: 0.5]: 0.2,[b: 0.25, c: 0.75]: 0.8]$

## Combining Lotteries

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$$
L_{1}=[a: 0.5, b: 0.5]
$$



$$
L_{2}=[b: 0.25, c: 0.75]
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## Combining Lotteries

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$$



$$
L_{2}=[b: 0.25, c: 0.75]
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## Combining Lotteries

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## Combining Lotteries


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Suppose that $L_{1}=[a: 0.5, b: 0.5]$ and $L_{2}=[b: 0.25, c: 0.75]$

- $\left[L_{1}: 0.2, L_{2}: 0.8\right]=[[a: 0.5, b: 0.5]: 0.2,[b: 0.25, c: 0.75]: 0.8]$ is a compound lottery
- The simplification of $\left[L_{1}: 0.2, L_{2}: 0.8\right]$ is $[a: 0.1, b: 0.3, c: 0.6]$.

Compound Lottery Axiom: The decision maker is indifferent between any lottery and its simplification.

## Two preferences

Suppose that $X=\{a, b\}$. Then the set of lotteries over $X$ is

$$
\mathcal{L}=\{[a: r, b:(1-r)] \mid 0 \leq r \leq 1\}
$$

1. Prefer lotteries that are closer to $50-50$ : E.g.,

$$
\left[a: \frac{1}{2}, b: \frac{1}{2}\right] P\left[a: \frac{1}{4}, b: \frac{3}{4}\right] I\left[a: \frac{3}{4}, b: \frac{1}{4}\right] P[a: 1, b: 0] I[a: 0, b: 1]
$$

2. Prefer lotteries with a higher chance of ending up with $a$ : E.g.,

$$
[a: 1, b: 0] P\left[a: \frac{3}{4}, b: \frac{1}{4}\right] P\left[a: \frac{1}{2}, b: \frac{1}{2}\right] P\left[a: \frac{1}{4}, b: \frac{3}{4}\right] P[a: 0, b: 1]
$$

The first preference violates the Independence Axiom while the second preference satisfies the Independence Axiom.

Independence: For all $L_{1}, L_{2}, L_{3} \in \mathcal{L}$ and $0<p \leq 1$,
$L_{1} P L_{2}$ if, and only if, $\left[L_{1}: p, L_{3}:(1-p)\right] P\left[L_{2}: p, L_{3}:(1-p)\right]$.
The following preferences violates independence:

$$
\left[a: \frac{1}{2}, b: \frac{1}{2}\right] P\left[a: \frac{1}{4}, b: \frac{3}{4}\right] I\left[a: \frac{3}{4}, b: \frac{1}{4}\right] P[a: 1, b: 0] I[a: 0, b: 1]
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$$

Let $L_{1}=\left[a: \frac{1}{2}, b: \frac{1}{2}\right], L_{2}=[a: 1, b: 0]$, and $L_{3}=[a: 0, b: 1]$.
Then, $L_{1} P L_{2}$.

Independence: For all $L_{1}, L_{2}, L_{3} \in \mathcal{L}$ and $0<p \leq 1$,
$L_{1} P L_{2}$ if, and only if, $\left[L_{1}: p, L_{3}:(1-p)\right] P\left[L_{2}: p, L_{3}:(1-p)\right]$.
The following preferences violates independence:

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\left[a: \frac{1}{2}, b: \frac{1}{2}\right] P\left[a: \frac{1}{4}, b: \frac{3}{4}\right] I\left[a: \frac{3}{4}, b: \frac{1}{4}\right] P[a: 1, b: 0] I[a: 0, b: 1]
$$

Let $L_{1}=\left[a: \frac{1}{2}, b: \frac{1}{2}\right], L_{2}=[a: 1, b: 0]$, and $L_{3}=[a: 0, b: 1]$.
Then, $L_{1} P L_{2}$.
However, since:

- $\left[L_{1}: \frac{1}{2}, L_{3}: \frac{1}{2}\right] I\left[a: \frac{1}{4}, b: \frac{1}{4}, a: 0, b: \frac{1}{2}\right]=\left[a: \frac{1}{4}, b: \frac{3}{4}\right]$
- $\left[L_{2}: \frac{1}{2}, L_{3}: \frac{1}{2}\right] I\left[a: \frac{1}{2}, b: \frac{1}{2}\right]$
we have: $\left[L_{2}: \frac{1}{2}, L_{3}: \frac{1}{2}\right] P\left[L_{1}: \frac{1}{2}, L_{3}: \frac{1}{2}\right]$.

Independence: For all $L_{1}, L_{2}, L_{3} \in \mathcal{L}$ and $0<p \leq 1$, $L_{1} P L_{2}$ if, and only if, $\left[L_{1}: p, L_{3}:(1-p)\right] P\left[L_{2}: p, L_{3}:(1-p)\right]$.

The following preferences violates independence:

$$
\left[a: \frac{1}{2}, b: \frac{1}{2}\right] P\left[a: \frac{1}{4}, b: \frac{3}{4}\right] I\left[a: \frac{3}{4}, b: \frac{1}{4}\right] P[a: 1, b: 0] I[a: 0, b: 1]
$$

Let $L_{1}=\left[a: \frac{1}{2}, b: \frac{1}{2}\right], L_{2}=[a: 1, b: 0]$, and $L_{3}=[a: 0, b: 1]$.
This violates independence since $L_{1} P L_{2}$ but it is not the case that $\left[L_{1}: \frac{1}{2}, L_{3}: \frac{1}{2}\right] P\left[L_{2}: \frac{1}{2}, L_{3}: \frac{1}{2}\right]$.

## Independence

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$$

## Independence




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For all $L, L^{\prime}, L^{\prime \prime} \in \mathcal{L}$ and $0<p \leq 1$,

$$
L P L^{\prime} \text { if, and only if, }\left[L: p, L^{\prime \prime}:(1-p)\right] P\left[L^{\prime}: p, L^{\prime \prime}:(1-p)\right] .
$$

$L I L^{\prime}$ if, and only if, $\left[L: p, L^{\prime \prime}:(1-p)\right] I\left[L^{\prime}: p, L^{\prime \prime}:(1-p)\right]$.

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$$

## Independence


 Arrow Rationality

A decision maker does not satisfy the Independence Axiom when there are lotteries $L_{1}, L_{2}, L_{3}$ and a number $p$ such that $0<p \leq 1$ such that at least one of the following is true:

1. $L_{1} P L_{2}$, but it is not the case that $\left[L_{1}: p, L_{3}:(1-p)\right] P\left[L_{2}: p, L_{3}:(1-p)\right]$;
2. $\left[L_{1}: p, L_{3}:(1-p)\right] P\left[L_{2}: p, L_{3}:(1-p)\right]$, but it is not the case that $L_{1} P L_{2}$;
3. $L_{1} I L_{2}$, but it is not the case that $\left[L_{1}: p, L_{3}:(1-p)\right] I\left[L_{2}: p, L_{3}:(1-p)\right]$; or
4. $\left[L_{1}: p, L_{3}:(1-p)\right] I\left[L_{2}: p, L_{3}:(1-p)\right]$, but it is not the case that $L_{1} I L_{2}$.
