PHPE 400
Individual and Group Decision Making

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A **rational** preference over lotteries involves more than the assumption that the decision maker's preferences are transitive and complete:

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2. Compound lottery axiom
3. Continuity axiom

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2. $L_2 \, P \, L_1$: The decision maker should strictly prefer L_2 to L_1.
3. $L_1 \, I \, L_2$: The decision maker should be indifferent between L_1 and L_2.
4. There is not enough information to answer this question.

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Suppose that the decision maker is rational and has the preference $a \, P \, b$ (the decision maker strictly prefers a to b) and c is another item.

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Suppose that the decision maker is rational and has the preference $a \, P \, b$ (the decision maker strictly prefers a to b) and c is another item.

Then, a *rational* decision maker will have the following preferences:

1. The decision maker strictly prefers $[a : 0.6, b : 0.4]$ over $[a : 0.4, b : 0.6]$
2. The decision maker strictly prefers $[a : 0.6, c : 0.4]$ over $[a : 0.6, c : 0.4]$

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Neither of these preferences can be inferred if all you know is that the decision maker's preferences over lotteries satisfies transitivity and completeness.

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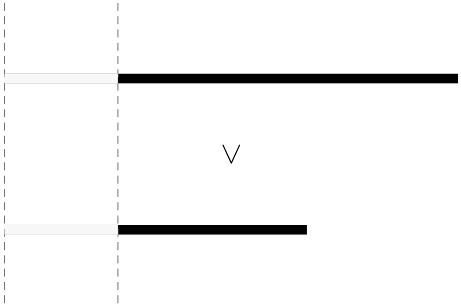

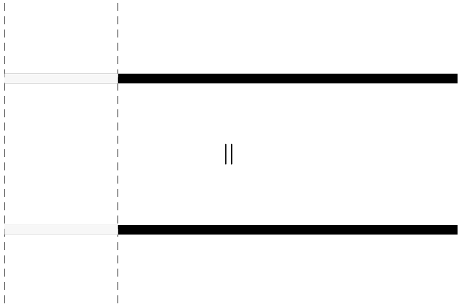

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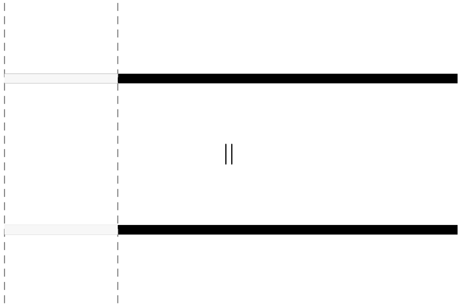

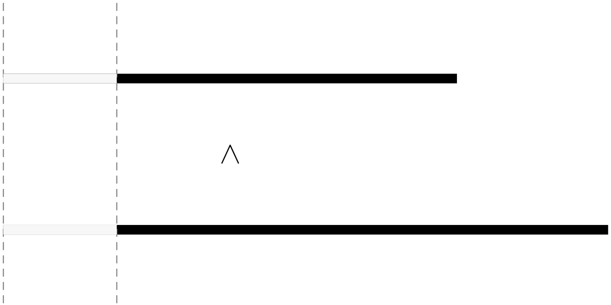

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Independence

For all $L_1, L_2, L_3 \in \mathcal{L}$ and $0 < p \leq 1$,

$$L_1 \; P \; L_2 \text{ if, and only if, } [L_1 : p,\; L_3 : (1-p)] \; P \; [L_2 : p,\; L_3 : (1-p)].$$

$$L_1 \; I \; L_2 \text{ if, and only if, } [L_1 : p,\; L_3 : (1-p)] \; I \; [L_2 : p,\; L_3 : (1-p)].$$

Independence

For all $\color{blue}\blacksquare$, $\color{red}\blacksquare$, $\color{green}\blacksquare$ $\in \mathcal{L}$ and $0 < p \leq 1$,

$$\color{blue}\blacksquare\ \color{black}P\ \color{red}\blacksquare \color{black}\text{ if, and only if, } [\color{blue}\blacksquare\color{black} : p,\ \color{green}\blacksquare\color{black} : (1-p)]\ P\ [\color{red}\blacksquare\color{black} : p,\ \color{green}\blacksquare\color{black} : (1-p)].$$

$$\color{blue}\blacksquare\ \color{black}I\ \color{red}\blacksquare \color{black}\text{ if, and only if, } [\color{blue}\blacksquare\color{black} : p,\ \color{green}\blacksquare\color{black} : (1-p)]\ I\ [\color{red}\blacksquare\color{black} : p,\ \color{green}\blacksquare\color{black} : (1-p)].$$

Combining Lotteries

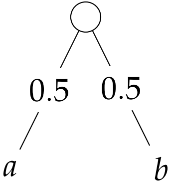
Suppose that $L_1 = [a : 0.5, b : 0.5]$ and $L_2 = [b : 0.25, c : 0.75]$

There are many ways to *combine* these lotteries.

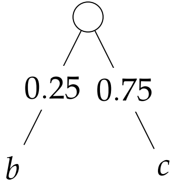
E.g., $[L_1 : 0.2, L_2 : 0.8] = [[a : 0.5, b : 0.5] : 0.2, [b : 0.25, c : 0.75] : 0.8]$

Combining Lotteries

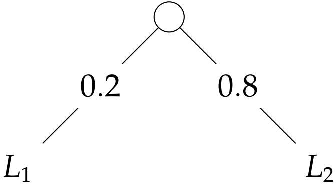
$L_1 = [a : 0.5, b : 0.5]$



$L_2 = [b : 0.25, c : 0.75]$

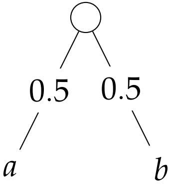


$[L_1 : 0.2, L_2 : 0.8]$

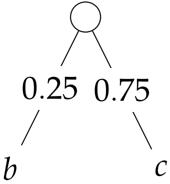


Combining Lotteries

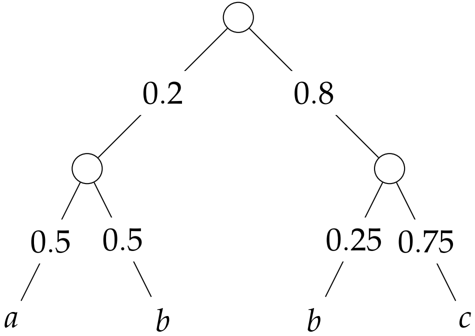
$$L_1 = [a : 0.5, b : 0.5]$$



$$L_2 = [b : 0.25, c : 0.75]$$



$$[L_1 : 0.2, L_2 : 0.8]$$



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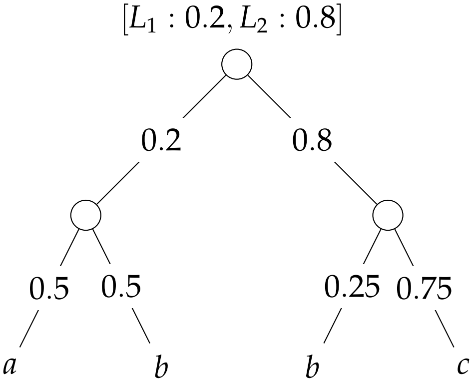

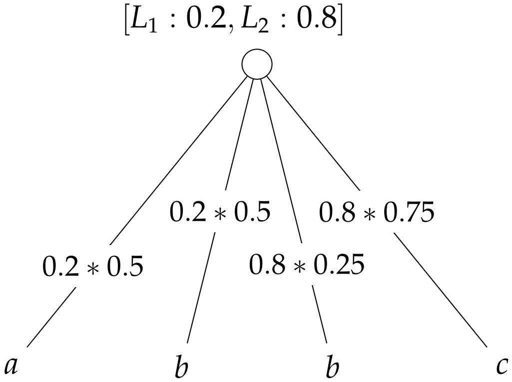

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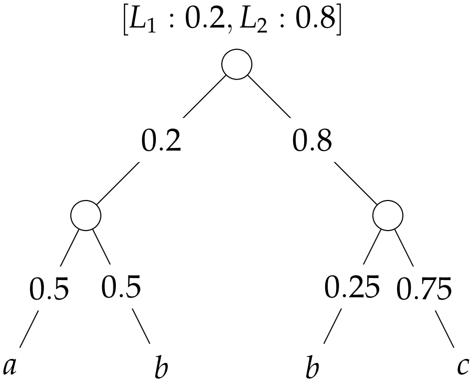
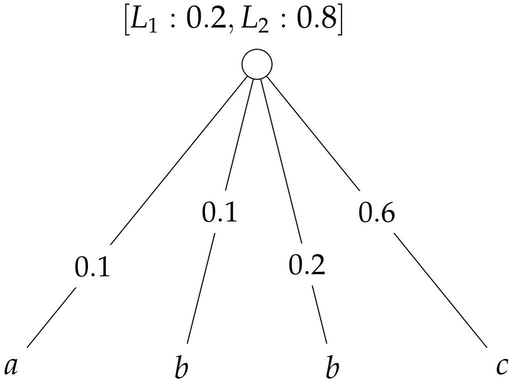
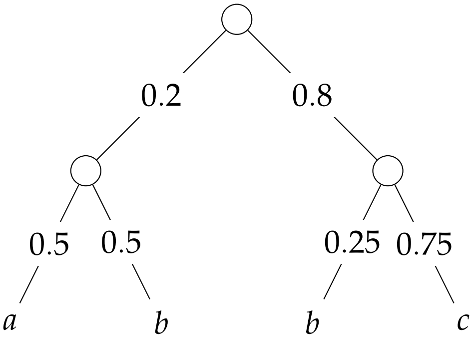




Combining Lotteries

$[L_1 : 0.2, L_2 : 0.8]$



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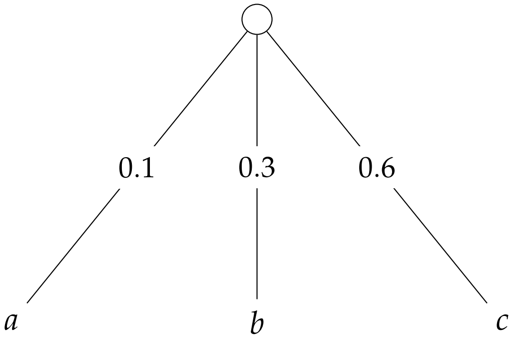

Combining Lotteries

Suppose that $L_1 = [a : 0.5, b : 0.5]$ and $L_2 = [b : 0.25, c : 0.75]$

- $[L_1 : 0.2, L_2 : 0.8] = [[a : 0.5, b : 0.5] : 0.2, [b : 0.25, c : 0.75] : 0.8]$ is a compound lottery
- The simplification of $[L_1 : 0.2, L_2 : 0.8]$ is $[a : 0.1, b : 0.3, c : 0.6]$.

Compound Lottery Axiom: The decision maker is indifferent between any lottery and its simplification.

Two preferences

Suppose that $X = \{a, b\}$. Then the set of lotteries over X is

$$\mathcal{L} = \{[a : r, b : (1 - r)] \mid 0 \leq r \leq 1\}$$

1. Prefer lotteries that are closer to 50-50: E.g.,

$$[a : \frac{1}{2}, b : \frac{1}{2}]\ P\ [a : \frac{1}{4}, b : \frac{3}{4}]\ I\ [a : \frac{3}{4}, b : \frac{1}{4}]\ P\ [a : 1, b : 0]\ I\ [a : 0, b : 1]$$

2. Prefer lotteries with a higher chance of ending up with a: E.g.,

$$[a : 1, b : 0]\ P\ [a : \frac{3}{4}, b : \frac{1}{4}]\ P\ [a : \frac{1}{2}, b : \frac{1}{2}]\ P\ [a : \frac{1}{4}, b : \frac{3}{4}]\ P\ [a : 0, b : 1]$$

The first preference violates the Independence Axiom while the second preference satisfies the Independence Axiom.

Independence: For all $L_1, L_2, L_3 \in \mathcal{L}$ and $0 < p \leq 1$,
$L_1 \; P \; L_2$ if, and only if, $[L_1 : p,\; L_3 : (1-p)] \; P \; [L_2 : p,\; L_3 : (1-p)]$.

The following preferences violates independence:

$$[a : \frac{1}{2}, b : \frac{1}{2}] \; P \; [a : \frac{1}{4}, b : \frac{3}{4}] \; I \; [a : \frac{3}{4}, b : \frac{1}{4}] \; P \; [a : 1, b : 0] \; I \; [a : 0, b : 1]$$

Independence: For all $L_1, L_2, L_3 \in \mathcal{L}$ and $0 < p \leq 1$,
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Let $L_1 = [a : \frac{1}{2}, b : \frac{1}{2}]$, $L_2 = [a : 1, b : 0]$, and $L_3 = [a : 0, b : 1]$.
Then, $L_1 \; P \; L_2$.

Independence: For all $L_1, L_2, L_3 \in \mathcal{L}$ and $0 < p \leq 1$,
$L_1 \ P \ L_2$ if, and only if, $[L_1 : p,\ L_3 : (1-p)] \ P \ [L_2 : p,\ L_3 : (1-p)]$.

The following preferences violates independence:

$$[a : \frac{1}{2}, b : \frac{1}{2}] \ P \ [a : \frac{1}{4}, b : \frac{3}{4}] \ I \ [a : \frac{3}{4}, b : \frac{1}{4}] \ P \ [a : 1, b : 0] \ I \ [a : 0, b : 1]$$

Let $L_1 = [a : \frac{1}{2}, b : \frac{1}{2}]$, $L_2 = [a : 1, b : 0]$, and $L_3 = [a : 0, b : 1]$.
Then, $L_1 \ P \ L_2$.

However, since:

- $[L_1 : \frac{1}{2}, L_3 : \frac{1}{2}] \ I \ [a : \frac{1}{4}, b : \frac{1}{4}, a : 0, b : \frac{1}{2}] = [a : \frac{1}{4}, b : \frac{3}{4}]$
- $[L_2 : \frac{1}{2}, L_3 : \frac{1}{2}] \ I \ [a : \frac{1}{2}, b : \frac{1}{2}]$

we have: $[L_2 : \frac{1}{2}, L_3 : \frac{1}{2}] \ P \ [L_1 : \frac{1}{2}, L_3 : \frac{1}{2}]$.

Independence: For all $L_1, L_2, L_3 \in \mathcal{L}$ and $0 < p \leq 1$,
$L_1 \; P \; L_2$ if, and only if, $[L_1 : p, \; L_3 : (1-p)] \; P \; [L_2 : p, \; L_3 : (1-p)]$.

The following preferences violates independence:

$$[a : \frac{1}{2}, b : \frac{1}{2}] \; P \; [a : \frac{1}{4}, b : \frac{3}{4}] \; I \; [a : \frac{3}{4}, b : \frac{1}{4}] \; P \; [a : 1, b : 0] \; I \; [a : 0, b : 1]$$

Let $L_1 = [a : \frac{1}{2}, b : \frac{1}{2}]$, $L_2 = [a : 1, b : 0]$, and $L_3 = [a : 0, b : 1]$.

This violates independence since $L_1 \; P \; L_2$ but it is not the case that $[L_1 : \frac{1}{2}, L_3 : \frac{1}{2}] \; P \; [L_2 : \frac{1}{2}, L_3 : \frac{1}{2}]$.

Independence

For all $L_1, L_2, L_3 \in \mathcal{L}$ and $0 < p \leq 1$,

$$L_1 \; P \; L_2 \text{ if, and only if, } [L_1 : p,\ L_3 : (1-p)] \; P \; [L_2 : p,\ L_3 : (1-p)].$$

$$L_1 \; I \; L_2 \text{ if, and only if, } [L_1 : p,\ L_3 : (1-p)] \; I \; [L_2 : p,\ L_3 : (1-p)].$$

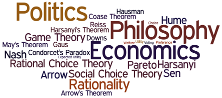
Independence

For all $L, L', L'' \in \mathcal{L}$ and $0 < p \leq 1$,

$$L \; P \; L' \text{ if, and only if, } [L : p, \; L'' : (1-p)] \; P \; [L' : p, \; L'' : (1-p)].$$

$$L \; I \; L' \text{ if, and only if, } [L : p, \; L'' : (1-p)] \; I \; [L' : p, \; L'' : (1-p)].$$

Independence



For all , ,  $\in \mathcal{L}$ and $0 < p \leq 1$,

 P  if, and only if, [ $: p$,  $: (1-p)$] P [ $: p$,  $: (1-p)$].

 I  if, and only if, [ $: p$,  $: (1-p)$] I [ $: p$,  $: (1-p)$].

Independence

A decision maker **does not** satisfy the Independence Axiom when there are lotteries L_1, L_2, L_3 and a number p such that $0 < p \leq 1$ such that at least one of the following is true:

1. $L_1 \; P \; L_2$, but it is not the case that $[L_1 : p, \; L_3 : (1-p)] \; P \; [L_2 : p, \; L_3 : (1-p)]$;
2. $[L_1 : p, \; L_3 : (1-p)] \; P \; [L_2 : p, \; L_3 : (1-p)]$, but it is not the case that $L_1 \; P \; L_2$;
3. $L_1 \; I \; L_2$, but it is not the case that $[L_1 : p, \; L_3 : (1-p)] \; I \; [L_2 : p, \; L_3 : (1-p)]$; or
4. $[L_1 : p, \; L_3 : (1-p)] \; I \; [L_2 : p, \; L_3 : (1-p)]$, but it is not the case that $L_1 \; I \; L_2$.