PHPE 400
Individual and Group Decision Making

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Rational Preferences

A pair (P, I) is a **rational preference** on X provided that $P \subseteq X \times X$ and $I \subseteq X \times X$, such that

- P is asymmetric and transitive. That is, P is a **strict weak order**.
- I is reflexive, symmetric, and transitive. That is, P is an **equivalence relation**.
- Completeness: For all $x, y \in X$, exactly one of $x \; P \; y$, $y \; P \; x$ or $x \; I \; y$ is true.

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Note that one need only define a strict preference relation P since I can be inferred assuming Completeness (e.g., if not-$x \; P \; y$ and not-$y \; P \; x$, then the decision maker must be indifferent between x and y).

Rational Choice

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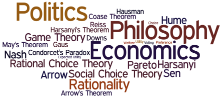
x is a maximal element of A with respect to P when there is no other element of A that is *strictly preferred* to y (i.e., there is no $y \in A$ such that $y \; P \; x$).

Utility Function

A **utility function** on a set X is a function $u : X \to \mathbb{R}$

Utility Function



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A rational preference (P, I) on X, is **represented** by a utility function $u : X \to \mathbb{R}$ if, and only if,

1. for all $x, y \in X$, $x\ P\ y$ when $u(x) > u(y)$
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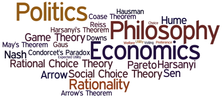
Ordinal Utility Theory



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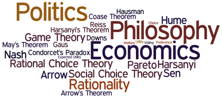
Utility is *defined* in terms of the decision maker's preference, **so it is an error to say that the decision maker prefers x to y *because* she assigns a higher utility to x than to y.**

Important



All three of the utility functions represent the preference $x \; P \; y \; P \; z$

Item	u_1	u_2	u_3
x	3	10	1000
y	2	5	99
z	1	0	1

$x \; P \; y \; P \; z$ is represented by both $(3, 2, 1)$ and $(1000, 999, 1)$, so one cannot say that y is “closer” to x than to z.

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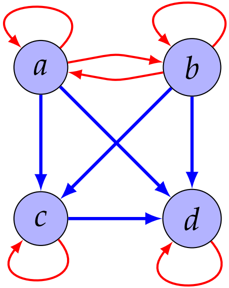

$$P = \{(a, c), (a, d), (c, d), (b, c), (b, d)\} \text{ and}$$
$$I = \{(a, a), (a, b), (b, a), (b, b), (c, c), (d, d)\}$$

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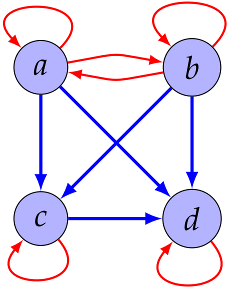

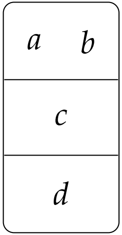

$$P = \{(a,c),(a,d),(c,d),(b,c),(b,d)\} \text{ and}$$
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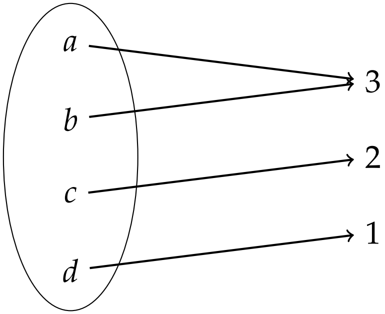

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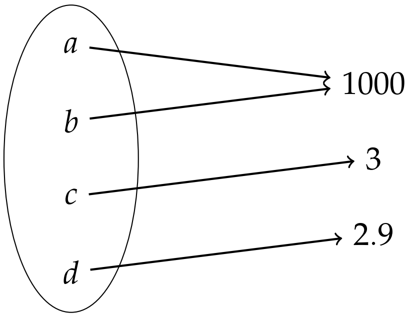

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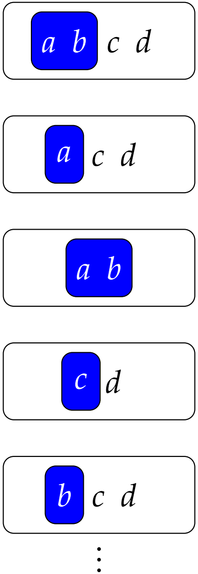
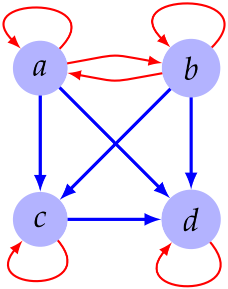
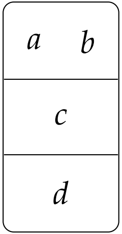
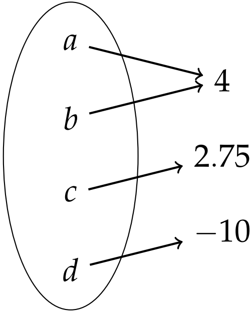
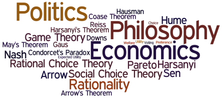








Decision under certainty



- The decision maker is certain about the consequence that will obtain given each of her available choices.

Feasible Options	Outcomes
a	o_1
b	o_2
c	o_3
$\vdots$	$\vdots$

Decision under risk

- The decision maker is certain about the probabilities associated with each consequence given each of her available choices.

Feasible Options	Outcomes
a	o_1 with probability p_1, o_2 with probability $p_2, \ldots$
b	o_1 with probability q_1, o_2 with probability $q_2, \ldots$
c	o_1 with probability r_1, o_2 with probability $r_2, \ldots$
$\vdots$	$\vdots$

Lotteries

Suppose that $X = \{x_1, \ldots, x_n\}$ is a set of outcomes.

A **lottery** over X is a tuple $[x_1 : p_1, \ldots, x_n : p_n]$ where $\sum_i p_i = 1$.

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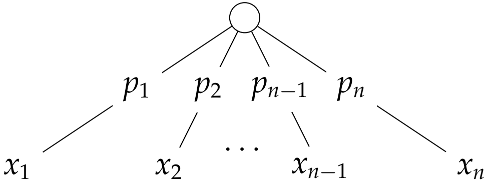

Expected Value of a Lottery

Suppose that the outcomes of a lottery are monetary values. So, $L = [x_1 : p_1, x_2 : p_2, \ldots, x_n : p_n]$, where each x_i is an amount of money. The **expected value** of L is:

$$\begin{aligned} EV([x_1 : p_1, \ldots, x_n : p_n]) &= p_1 \times x_1 + \cdots + p_n \times x_n \\ &= \textstyle\sum_{i=1}^{n} p_i \times x_i \end{aligned}$$

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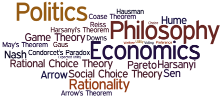

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E.g., if $L = [\$100 : 0.55, \$50 : 0.25, \$0 : 0.20]$, then

$$EV(L) = 0.55 * 100 + 0.25 * 50 + 0.2 * 0 = 67.5$$

You are given a choice between two lotteries L_1 and L_2. The outcome of the lotteries is determined by flipping a fair coin. The payoff for the two lotteries are given in the following table:

	Heads	Tails
L_1	\$1M	\$1M
L_2	\$3M	\$0

Which of the two lotteries would you choose?

1. L_1
2. L_2
3. I am indifferent between the two lotteries