PHPE 400
Individual and Group Decision Making

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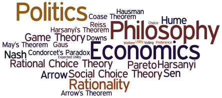
pacuit.org

Preferences - Minimal Constraints



A decision maker's preferences on X is represented by three relations $P \subseteq X \times X$, $I \subseteq X \times X$ and $N \subseteq X \times X$ satisfying the following minimal constraints:

1. For all $x, y \in X$, exactly one of $x \; P \; y$, $y \; P \; x$, $x \; I \; y$ and $x \; N \; y$ is true.
2. P is asymmetric
3. I is reflexive and symmetric.
4. N is symmetric.

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Transitive relation: for all $x, y, z \in X$, if $x \mathrel{R} y$ and $y \mathrel{R} z$, then $x \mathrel{R} z$

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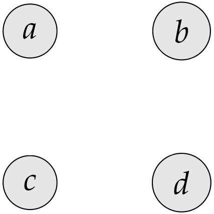

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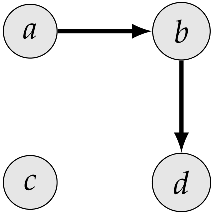

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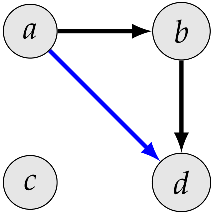

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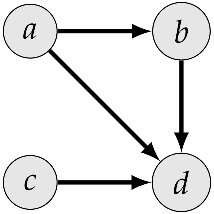

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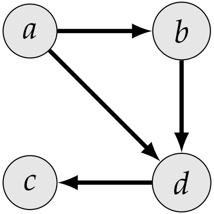

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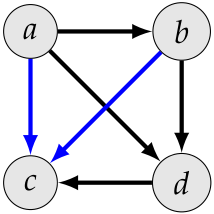

Transitivity

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? Indifference is transitive: for all x, y, z if $x \; I \; y$ and $y \; I \; z$ then $x \; I \; z$

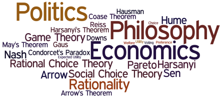
? Non-comparability is transitive: for all x, y, z if $x \; N \; y$ and $y \; N \; z$ then $x \; N \; z$.

Transitivity

Indifference: For all $x, y, z \in X$, if $x\ I\ y$ and $y\ I\ z$, then $x\ I\ z$.

- You may be indifferent between a curry with x amount of cayenne pepper, and a curry with x plus one particle of cayenne pepper for any amount x. But you are not indifferent between a curry with no cayenne pepper and one with 1 pound of cayenne pepper in it!

Transitivity



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Incomparibility: For all $x, y, z \in X$, if xNy and yNz, then xNz.

- You may not be able to compare having a job as a teacher with having a job as lawyer. Furthermore, you cannot compare having a job as a lawyer with having a job as a teacher with an extra $1,000. However, you do strictly prefer having a job as a teacher with an extra $1,000 to having a job as a teacher.

Transitivity

There are two ways that a decision maker's strict preference P on X may fail transitivity:

1. The decision maker lacks a strict preference: There are $x, y, z \in X$ such that xPy and yPz, but xNz (i.e., x and z are incomparable).
2. There is a *cycle* in the decision maker's preferences: There are $x, y, z \in X$ such that xPy, yPz, and zPx.

Cyclic Preferences

I do not think we can clearly say what should convince us that [someone] at a given time (without change of mind) preferred *a* to *b*, *b* to *c* and *c* to *a*. The reason for our difficulty is that we cannot make good sense of an attribution of preference except against a background of coherent attitudes...My point is that if we are intelligibly to attribute attitudes and beliefs, or usefully to describe motions as behaviour, then we are committed to finding, in the pattern of behaviour, belief, and desire, a large degree of rationality and consistency. (Davidson 1974: p. 237)

D. Davidson. *'Philosophy as psychology'*. In S. C. Brown (ed.), Philosophy of Psychology, 1974. Reprinted in his Essays on Actions and Events. Oxford: OUP 2001: pp. 229–244.

Money-Pump Argument

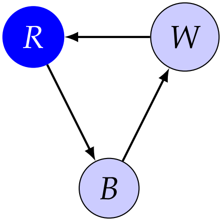
There are three key assumptions about a decision maker's strict preference P and the decision maker's opinion about money:

1. If xPy, then the decision maker will always take x when y is the only alternative.

2. If xPy, then there is some $v > 0$ such that for all u, $(x, -\$u)Py$ if and only if $0 \leq u \leq v$.

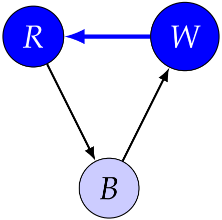
3. The items and money are *separable* and the decision maker prefers more money to less: For all $x, y \in X$ and $w, z \in \mathbb{R}$, we have that
   - $(x, \$w)P(x, \$z)$ if and only if $w > z$; and,
   - if xPy, then $(x, \$w)P(y, \$w)$.

Money-Pump Argument



(R)

Money-Pump Argument



$$(R) \implies (W, -1)$$

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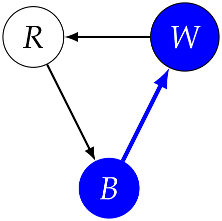

$$(R) \implies (W, -1) \implies (B, -2)$$

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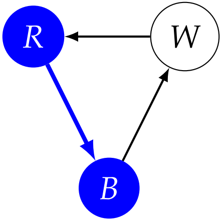

$$(R) \implies (W, -1) \implies (B, -2) \implies (R, -3) \implies (W, -4) \implies \cdots$$

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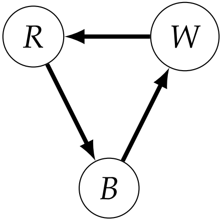

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✗ There is a *cycle* in the decision maker's preferences: There are $x, y, z \in X$ such that xPy, yPz, and zPx.

$\Rightarrow$ Money-pump argument, rankings, ...

Completeness

Completeness: For all $x, y \in X$, exactly one of $x \; P \; y$, $y \; P \; x$ or $x \; I \; y$ is true. I.e., for all $x, y \in X$, not-$x \; N \; y$.

Completeness

> To have complete and transitive preferences over such complex alternatives requires more knowledge than anyone is likely to have.
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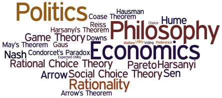

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[O]f all the axioms of utility theory, the completeness axiom is perhaps the most questionable. Like others, it is inaccurate as a description of real life; but unlike them we find it hard to accept even from the normative viewpoint.

(Aumann, 1962)

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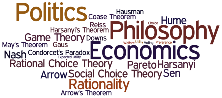
Rational Preferences



A pair (P, I) is a **rational preference** on X provided that $P \subseteq X \times X$ and $I \subseteq X \times X$, such that

- P is asymmetric and transitive. That is, P is a **strict weak order**.
- I is reflexive, symmetric, and transitive. That is, P is an **equivalence relation**.
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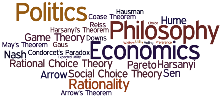

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Note that one need only define a strict preference relation P since I can be inferred assuming Completeness (e.g., if not-x P y and not-y P x, then the decision maker must be indifferent between x and y).