PHPE 400
Individual and Group Decision Making

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Evaluative Voting

In Arrow's theorem, it is assumed that the input is the voters' *rankings* of the candidates.

One response to Arrow's theorem is to ask for more information from the voters about their opinions of the candidates.

https://electionscience.org

Approval Voting: Each voter selects a subset of candidates. The candidate with the most "approvals" wins the election.

S. Brams and P. Fishburn. *Approval Voting*. Birkhauser, 1983.

J.-F. Laslier and M. R. Sanver (eds.). *Handbook of Approval Voting*. Studies in Social Choice and Welfare, 2010.

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Approving of a candidate is not (necessarily) the same as simply ranking the candidate first.

Why Approval Voting?

https://electionscience.org

S. Brams and P. Fishburn. *Going from Theory to Practice: The Mixed Success of Approval Voting*. Handbook of Approval Voting, pp. 19-37, 2010.

Approval Voting is more flexible

# voters	2	2	1
	a	*b*	*c*
	d	*d*	*a*
	b	*a*	*b*
	c	*c*	*d*

The Condorcet winner is *a*.

Approval Voting is more flexible

There is no fixed rule that always elects a unique Condorcet winner.

# voters	2	2	1
	a	b	c
	d	d	a
	b	a	b
	c	c	d

The Condorcet winner is a.
Vote-for-1 elects $\{a, b\}$

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# voters	2	2	1
	a	b	c
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	c	c	d

The Condorcet winner is a.
Vote-for-1 elects $\{a, b\}$, vote-for-2 elects $\{d\}$

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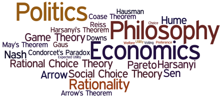

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# voters	2	2	1
	a	b	c
	d	d	a
	b	a	b
	c	c	d

The Condorcet winner is a.
Vote-for-1 elects $\{a, b\}$, vote-for-2 elects $\{d\}$, vote-for-3 elects $\{a, b\}$.

Approval Voting is more flexible

AV may elect the Condorcet winner

# voters	2	2	1
	a	b	c
	d	d	a
	b	a	b
	c	c	d

The Condorcet winner is a.
$(\{a\}, \{b\}, \{c, a\})$ elects a under AV.

Possible Failure of Unanimity

# voters	1	1	1
	a	*c*	*d*
	b	*a*	*a*
	c	*b*	*b*
	d	*d*	*c*

Possible Failure of Unanimity

# voters	1	1	1
	a	c	d
	b	a	a
	c	b	b
	d	d	c

Approval Winners: a, b

Generalizing Approval Voting

In many group decision situations, people use measures or grades from a **common language of evaluation** to evaluate candidates or alternatives:

- in figure skating, diving and gymnastics competitions;
- in piano, flute and orchestra competitions;
- in classifying wines at wine competitions;
- in ranking university students;
- in classifying hotels and restaurants, e.g., the Michelin $*$

Score Voting/Range Voting

Governor Candidates		Score *each* candidate by filling a number (0 is worst; 9 is best)
1: Candidate A	→	**0** 1 2 3 4 5 6 7 8 9
2: Candidate B	→	0 1 2 3 4 5 6 7 8 **9**
3: Candidate C	→	0 1 2 3 4 5 6 **7** 8 9

Score Voting/Range Voting

Fixe a common grading language consisting of, for example, the integers $\{0, 1, 2, \ldots, 10\}$

The candidate with the largest *average* grade is declared the winner.

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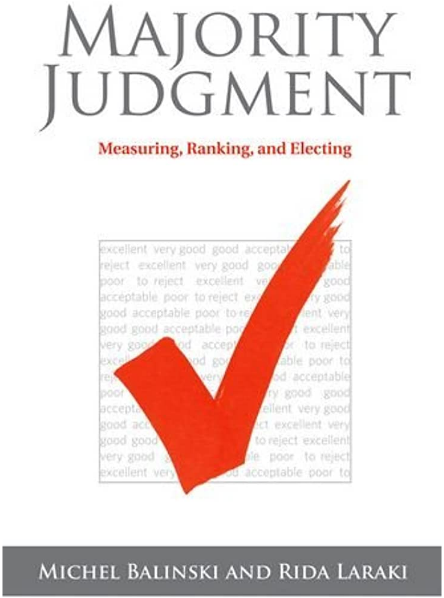
The candidate with the largest *average* grade is declared the winner.

Suppose a's grades are $\{7, 7, 8, 8, 9, 9, 9, 10\}$. The average grade is 8.375

Suppose b's grades are $\{9, 9, 9, 9, 9, 10, 10, 10\}$. The average grade is 9.375

So, Score Vote (Range Vote) ranks b above candidate a.


MAJORITY JUDGMENT
Measuring, Ranking, and Electing
MICHEL BALINSKI AND RIDA LARAKI

Majority Judgment

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1. Each voter assigns each candidate a score from the set $\{0, \ldots, n\}$;
2. The candidate with the greatest median score wins according to Majority Judgement.

Note: if there is an even number of voters, Majority Judgement uses the "lower median." E.g., if the scores for A are $7, 7, 8, 8, 11, 11, 11, 13$, the lower median is 8.

Evaluative Voting

Approval Voting: voters can assign a single grade "approve" to the candidates. The candidates with the most approvals are the winner.

Score Voting: voters can assign any grade from a fixed set of grades to the candidates. The candidate with the greatest sum of the scores is the winner.

Majority Judgement: voters can assign any grade from a fixed set of grades to the candidates. The candidate with the greatest median score is the winner.

Score Voting vs. Majority Judgement

Consider the following example from the SEP entry on "Voting Methods":

# of Voters	A	B	C
1	4	3	1
1	4	3	2
1	2	0	3
1	2	3	4
1	1	0	2
Mean:	2.6	1.8	2.4
Median:	2	3	2

Score Voting vs. Majority Judgement

Consider the following example from the SEP entry on "Voting Methods":

# of Voters	A	B	C
1	4	3	1
1	4	3	2
1	2	0	3
1	2	3	4
1	1	0	2
Mean:	2.6	1.8	2.4
Median:	2	3	2

Thus, *A* wins according to Score Voting, while *B* wins according to Majority Judgement.

Score Voting vs. Majority Judgement

Here is another example from the *Majority Voting* book (p. 282) showing how Majority Judgement differs from Score Voting:

# of Voters	A	B
k	20	20
1	19	20
k	19	0
Mean:	slightly under 19.5	slightly over 10
Median:	19	20

Grading vs. Ranking

S. Brams and R. Potthoff. *The paradox of grading systems*. Public Choice, 165, pp. 193 - 210, 2015.

Grading vs. Ranking

Suppose that the possible grades are $\{0, 1, \ldots, 20\}$

# of Voters	A	B
1	20	11
1	9	0
1	9	10
Median:	9	10

Majority Judgement Winner: B

Grading vs. Ranking

Suppose that the possible grades are $\{0, 1, \ldots, 20\}$

# of Voters	A	B
1	20	11
1	9	0
1	9	10
Median:	9	10

Majority Judgement Winner: B

2 out of 3 voters prefer A to B

Grading vs. Ranking

Suppose that the possible grades are $\{0, 1, \ldots, 20\}$

# of Voters	A	B
50	20	11
50	9	0
1	9	10
Median:	9	10

Majority Judgement Winner: B

Grading vs. Ranking

Suppose that the possible grades are $\{0, 1, \ldots, 20\}$

# of Voters	A	B
50	20	11
50	9	0
1	9	10
Median:	9	10

Majority Judgement Winner: B

100 out of 101 voters prefer A to B

Grades: $\{0, 1, 2, 3, 4, 5\}$
Candidates: $\{A, B, C\}$
5 Voters

# of Voters	A	B	C
1	5	0	0
4	0	1	1
Mean:	1	4/5	4/5

Grades: $\{0, 1, 2, 3, 4, 5\}$
Candidates: $\{A, B, C\}$
5 Voters

# of Voters	A	B	C
1	5	0	0
4	0	1	1
Mean:	1	4/5	4/5

Average Grade Winner: A

Superior Grade Winner: B, C

To conclude, we have identified a paradox of grading systems, which is not just a mirror of the well-known differences that crop up in aggregating votes under ranking systems. Unlike these systems, for which there is no accepted way of reconciling which candidate to choose when, for example, the Hare, Borda and Condorcet winners differ, AV provides a solution when the AG and SG winners differ.

Theorem (Brams and Potthoff). When there are two grades, the AG and SG winners are identical.

The Preference Intensity Problem

51	49
a	b
b	a

51% of the voters have a *slight* preference for a over b and 49% of the voters have a *strong* preference for b over a.

Should candidate a win the election?

The Preference Intensity Problem

80	20
a	b
b	a

80% of the voters *strictly prefer* a over b and 20% of the voters have an "*extremely strong*" preference for b over a.

Should candidate a win the election?

The Preference Intensity Problem

75	25
a	b
b	a

75% of the voters *strictly prefer* a over b and 25% of the voters *strictly prefer* b over a. If a wins, then this will cause harm to the 25% of voters that prefer b to a; and if b wins, this will cause some annoyance to the 75% of the voters that prefer a to b.

How do we weigh the preference of the majority while avoiding harm to the minority?

The Preference Intensity Problem

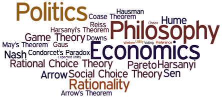
75	25
a	b
b	a

75% of the voters *strictly prefer* a over b and 25% of the voters *strictly prefer* b over a. If a wins, then this will cause harm to the 25% of voters that prefer b to a; and if b wins, this will cause some annoyance to the 75% of the voters that prefer a to b.

How do we weigh the preference of the majority while avoiding harm to the minority?

- Not all questions should be decided by a vote.
- Education, deliberation, etc. to change the rankings of the enough of the 75% of the voters to ensure that b is the majority opinion.

Systematic Minority



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- If voters cast a single vote for a single candidate, the majority, no matter how slender, is guaranteed victory.
- When group barriers are permeable, the minority can occasionally belong to the winning side.
- When preferences are fully polarized and the power of a cohesive majority bloc is secure, the minority remains disenfranchised.
- Some solutions:
  - Ensure that the political districts are *fair*: `https://mggg.org/`
  - In some instances power-sharing is imposed directly, and the constitution grants executive positions to specific groups, typically on the basis of their ethnic or religious identity. The problem is that constitutional provisions of this type are difficult to enforce and heavy-handed.

Storable Votes

In a setting with a finite number of binary issues, the Storable Votes mechanism grants a fixed number of total votes to each voter with the freedom to divide them as wished over the different issues, knowing that each issue will be decided by simple majority.

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- Storable Votes allows the minority to prevail occasionally and yet is anonymous and treats everyone identically.
- Storable Votes can apply to direct democracy in large electorates, or to smaller groups, possibly legislatures or committees formed by voters' representatives.

A. Casella (2005). *Storable votes*. Games and Economic Behavior, 51(2), pp. 391 - 419.

A. Casella (2012). *Storable votes: Protecting the minority voice*. Oxford: Oxford University Press.

Although easy to describe, Storable Votes poses a challenging strategic problem: how should a voter best divide her votes over the different issues? Note a central ingredient of the strategic environment: the hide-and-seek nature of the game between majority and minority voters. If the majority spreads its votes evenly, then the minority can win some issues by concentrating its votes on them, but if the majority knows in advance which issues the minority is targeting, then the majority can win those too.

A. Casella, J.-F. Laslier, and A. Macé. *Democracy for Polarized Committees: The Tale of Blotto's Lieutenants*. Games and Economic Behavior, 106, pp. 239 - 225, 2017.

Grades vs. Utilities

The language of grades has nothing to do with utilities (viewed as measures of individual satisfaction).

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The language of grades has nothing to do with utilities (viewed as measures of individual satisfaction). Grades are absolute measures of merit. In the context of voting and judging, utilities are relative measures of satisfaction. (Balinksi and Laraki, p. 185)

Utility Functions

A **utility function** on a set X is a function $u : X \to \mathbb{R}$

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A preference ordering is **represented** by a utility function iff x is (weakly) preferred to y provided $u(x) \geq u(y)$

L. Narens and B. Skyrms . *The Pursuit of Happiness Philosophical and Psychological Foundations of Utility*. Oxford University Press, 2020.

Let X and V be nonempty sets with $|X| \geq 3$ and V finite.

Let $\mathcal{U}(X)$ be the set of all functions $u : X \to \mathbb{R}$

A **profile** is a function $\mathbf{U} : V \to \mathcal{U}(X)$, write $\mathbf{U}_i$ for voter i's utility function on X in profile $\mathbf{U}$.

A **Social Welfare Functional (SWFL)** is a function f mapping profiles of utilities to asymmetric relations on X. So for each profile $\mathbf{U}$, $f(\mathbf{U})$ is the social preference order on X.

Sum Utilitarian: Define f_S as follows: For all $x, y \in X$,

$$x \; f_S(\mathbf{U}) \; y \text{ if and only if } \sum_i \mathbf{U}_i(x) \geq \sum_i \mathbf{U}_i(y)$$

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Maximin: Define f_M as follows: For all $x, y \in X$,

$$x \, f_M(\mathbf{U}) \, y \text{ if and only if } \min_i \{\mathbf{U}_i(x)\} \geq \min_i \{\mathbf{U}_i(y)\}$$

U	x	y	z
a	3	1	8
b	3	2	1
c	1	4	1

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a	3	1	8
b	3	2	1
c	1	4	1
Sum	7	7	10
Min	1	1	1

- Sum utilitarian: z is ranked above x and y, and x and y are tied.
- Maximin: x, y and z are all tied.

$\mathbf{U}$	x	y	z
a	1	1	8
b	3	1	1
c	5	4	1
Min	1	1	1

Strong Pareto: For all $x, y \in X$, if $\mathbf{U}_i(x) \geq \mathbf{U}_i(y)$ for all $i \in V$ and there is a $j \in V$ such that $U_j(x) > U_j(y)$, then $f(\mathbf{U})$ must rank x strictly above y.

- Maximin violates strong Pareto: x, y and z are all tied; however, shouldn't x be ranked above y?

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- Lexicographic Maximin: rank x above y when $\min_i\{\mathbf{U}_i(x)\} \geq \min_i\{\mathbf{U}_i(y)\}$, breaking ties lexicographically.

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  Lexicographic Maximin: x is ranked above z and z is ranked above y:

$$\langle 1, 3, 5\rangle > \langle 1, 1, 8\rangle > \langle 1, 1, 4\rangle$$

U	x	y	z
a	3	1	8
b	3	2	1
c	1	4	1
Sum	7	7	10
Min	1	1	1

- Sum utilitarian: z is ranked above x and y, and x and y are tied.
- ~~Maximin: x, y and z are all tied.~~
- Lexicographic Maximin: x is ranked above y, and y is ranked above z.

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Lexicographic Maximin: Define f_M as follows: For all $x, y \in X$,

$$x\, f_{LM}(\mathbf{U})\, y \text{ if and only if } \min_i\{\mathbf{U}_i(x)\} \geq \min_i\{\mathbf{U}_i(y)\}$$

breaking ties lexicographically: e.g., $\langle 9, 3, 1, 2\rangle$ is "less than" $\langle 1, 2, 4, 8\rangle$.

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Both SWFLs satisfy versions of Arrow's axioms, including non-dictatorship!

Arrow Axioms

Transitivity/Completeness: For all **U** in the domain of f, $f(\mathbf{U})$ is transitive/complete.

Universal Domain: the domain of f is the set of all profiles

Weak Pareto: For all **U** in the domain of f, for all $x, y \in X$, if $\mathbf{U}_i(x) > \mathbf{U}_i(y)$ for all $i \in V$, then x is ranked strictly above y according to $f(\mathbf{U})$.

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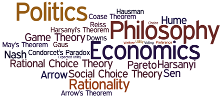

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Independence of Irrelevant Utilities: For all $\mathbf{U}$ and $\mathbf{U}'$ in the domain of f, for all $x, y \in X$, if $\mathbf{U}_i(x) = \mathbf{U}'_i(x)$ and $\mathbf{U}_i(y) = \mathbf{U}'_i(y)$ for all $i \in V$, then $x \, f(\mathbf{U}) \, y$ if and only if $x \, f(\mathbf{U}') \, y$.