PHPE 400
Individual and Group Decision Making

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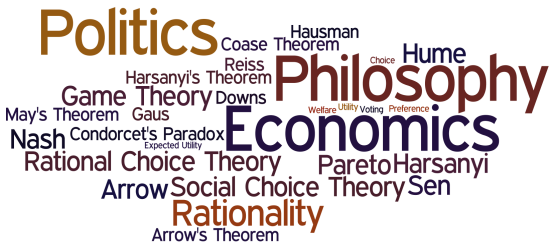

May's Theorem is a *proceduralist* justification of majority rule showing that Majority Rule is the unique group decision method satisfying two basic principles of fairness (Anonymity and Neutrality) and a basic principle ensuring that the outcome responds appropriately to the voters' opinions (Weak Positive Responsiveness).

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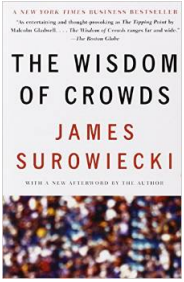
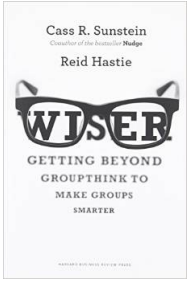
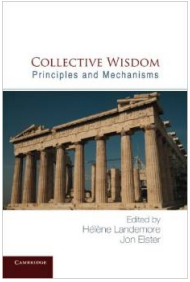
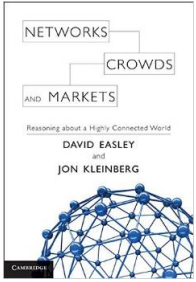
The Condorcet Jury Theorem is an *epistemic* justification of majority rule showing that under the assumption that the voters are *competent* in the sense that each voters has a greater than 50% chance of voting correctly and that the events that the voters are correct are independent, then the probability that the majority is correct increases to 1 as the size of the group increases.

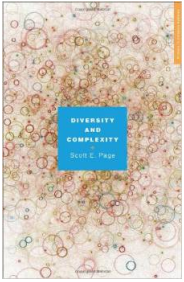
Collective Intelligence











Can May's Theorem be generalized to more than 2 candidates?

Can May's Theorem be generalized to more than 2 candidates? No!

- Group decision problems often exhibit a *combinatorial structure*. For example, voting on a number of yes/no issues in a referendum, or voting on different interconnected issues, or selecting a committee from a set of candidates.
- As we have seen, there are many reasonable voting methods that generalize Majority Rule for more than 2 candidates.

Multiple Elections Paradox

S. Brams, D. M. Kilgour, and W. Zwicker. *The paradox of multiple elections*. Social Choice and Welfare, 15(2), pgs. 211 - 236, 1998.

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

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YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: N (7 - 6)

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: N (7 - 6)
Proposition 2: N (7 - 6)

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: N (7 - 6)
Proposition 2: N (7 - 6)
Proposition 3: N (7 - 6)

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	*YYN*	*YNY*	*YNN*	*NYY*	*NYN*	*NNY*	*NNN*
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: *N* (7 - 6)
Proposition 2: *N* (7 - 6)
Proposition 3: *N* (7 - 6)

But there is no support for NNN!

S. Brams, M. Kilgour and W. Zwicker. *Voting on referenda: the separability problem and possible solutions*. Electoral Studies, 16(3), pp. 359 - 377, 1997.

D. Lacy and E. Niou. *A problem with referenda*. Journal of Theoretical Politics 12(1), pp. 5 - 31, 2000.

J. Lang and L. Xia. *Sequential composition of voting rules in multi-issue domains*. Mathematical Social Sciences 57(3), pp. 304 - 324, 2009.

L. Xia, V. Conitzer and J. Lang. *Strategic Sequential Voting in Multi-Issue Domains and Multiple-Election Paradoxes*. In Proceedings of the Twelfth ACM Conference on Electronic Commerce (EC-11), pp. 179-188, 2010.

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"Is a conflict between the proposition and combination winners necessarily bad? ... The paradox does not just highlight problems of aggregation and packaging, however, but strikes at the core of social choice—both what it means and how to uncover it. In our view, the paradox shows there may be a clash between two different meanings of social choice, leaving unsettled the best way to uncover what this elusive quantity is." (pg. 234).

S. Brams, D. M. Kilgour, and W. Zwicker. *The paradox of multiple elections*. Social Choice and Welfare, 15(2), pgs. 211 - 236, 1998.

Judgement Aggregation Paradoxes

Kornhauser and Sager. *Unpacking the court*. Yale Law Journal, 1986.

P. Mongin. *The doctrinal paradox, the discursive dilemma, and logical aggregation theory*. Theory and Decision, 73(3), pp 315 - 355, 2012.

C. List and P. Pettit. *Aggregating sets of judgments: An impossibility result*. Economics and Philosophy 18, pp. 89 - 110, 2002.

Judgement Aggregation Paradox

Should we hire the candidate?

- Is the candidate good at research (r)?
- Is the candidate good at teaching (t)?
- We should hire the candidate if and only if the candidate is good at research and teaching. ($r \wedge t$)

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

	r	t		h
Voter 1				
Voter 2				
Voter 3				
Group				

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

	r	t		h
Voter 1	Yes	Yes		
Voter 2	Yes	No		
Voter 3	No	Yes		
Group	Yes	Yes		

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes		
Voter 2	Yes	No		
Voter 3	No	Yes		
Group	Yes	Yes	Yes	Yes

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes	Yes	Yes
Voter 2	Yes	No	Yes	No
Voter 3	No	Yes	Yes	No
Group				No

Judgement Aggregation Paradox

Is the candidate good at research (r)? Is the candidate good at teaching (t)?
Should we hire the candidate (h)?

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes	Yes	Yes
Voter 2	Yes	No	Yes	No
Voter 3	No	Yes	Yes	No
Group	Yes	Yes	Yes	Y/N