PHPE 400
Individual and Group Decision Making

Eric Pacuit
University of Maryland
pacuit.org

Example

Voting Methods: Plurality, Borda, Instant Runoff Voting (Ranked Choice Voting), Coombs, Minimax, Copeland, Split Cycle

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- Voting methods that satisfy the top condition (winners must be ranked first by at least one voter): Plurality and Instant Runoff Voting (Ranked Choice Voting)
- Voting methods that always elect a Condorcet winner (when one exists): Minimax, Copeland, Split Cycle

Which Voting Method is Best?

A 2004 letter to the Washington Post sent by a local organizer of the Green Party, as quoted by Miller (2019, p. 119):

> *[Electoral engineering] isn't rocket science. Why is it that we can put a man on the moon but can't come up with a way to elect our president that allows voters to vote for their favorite candidate, allows multiple candidates to run and present their issues and… [makes] the 'spoiler' problem… go away?*

The Spoiler Problem

2,912,790	2,912,253	97,488
Bush	Gore	Nader
Gore	Nader	Gore
Nader	Bush	Bush

Nader *spoiled* the election for Gore.

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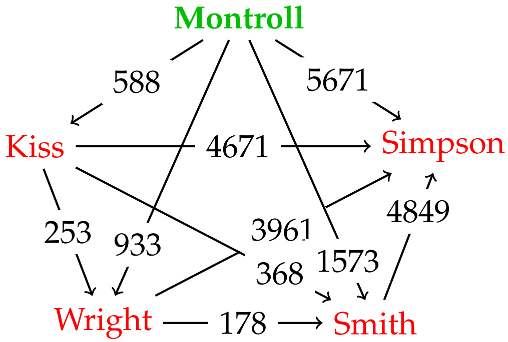

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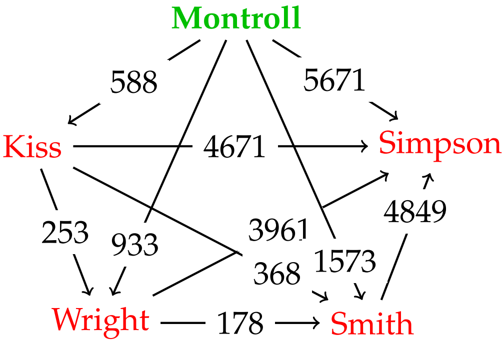

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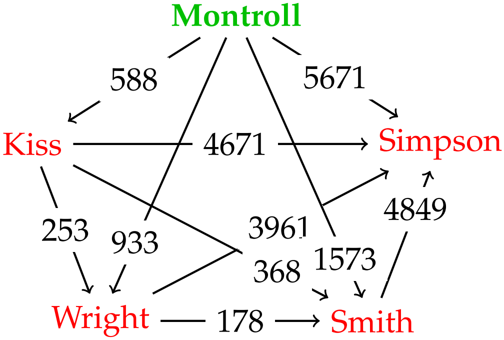

Montroll was the **Condorcet winner**. IRV was repealed in 2010.

The Spoiler Problem

37	29	34
d	d	p
p	p	d

Ranked Choice winner:
d

37	29	34
r	d	p
d	p	d
p	r	r

Ranked Choice winner:
p

r spoils the election for d: A majority prefers d to r, but the addition of r knocks d out of the winning set.

(From www.electionscience.org/library/the-spoiler-effect/:
"a simplified approximation of what happened in the 2009 IRV mayoral election in Burlington, Vermont.")

Examples of spoiler effects

- **2000 Florida Presidential Election** (Plurality):

  Gore would have won had the election not included Nader, whom Gore (plausibly) beat head-to-head. But with Nader included, Bush won.

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- **2007 Burlington Mayoral Election** (Instant Runoff):

  Montroll would have won had the election not included Wright, whom Montroll beat head-to-head. But with Wright included, Kiss won.

- **2022 Special Election for U.S. Rep. in Alaska** (Instant Runoff):

  Begich would have won had the election not included Palin, whom Begich beat heat-to-head. But with Palin included, Peltola won.

Immunity to Spoilers

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This criterion rules out all the spoiler effects we've discussed.

But do any useable voting methods satisfy it—or is it too good to be true?

Broda

Borda violates Immunity to Spoilers:

2	3
c	a
a	c

$$a \xrightarrow{1} c$$

Borda winner: a

2	3
c	a
b	c
a	b

$$\begin{tikzcd} a \arrow[r, "1"] \arrow[rr, bend left, "1"] & c \arrow[r, "5"] & b \end{tikzcd}$$

Borda winner: c

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Let **P** be the election on the right.

- a is a Borda winner without b in the election **P**
- a is majority preferred to b in **P**
- a and b both lose in **P** according to Borda

The only voting methods that you have seen so far that satisfy Immunity to Spoilers is Minimax, Copeland and Split Cycle.

What other properties do we want a voting method to satisfy?

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Can we use these properties to *characterize* voting methods?

Characterizing Majority Rule

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Majority Rule: a is ranked above (below) b if more (fewer) voters rank a above b than b above a, otherwise a and b are tied.

When there are only two options, can we argue that majority rule is the "best" procedure?

Democracy: The decisions made by a group must be appropriately responsive to the expressed wishes of the members of that group.

Political equality: Each group member must have an equal (chance of) influence over the group's decisions.

Majority rule: The option that gets the most votes should be the group decision.

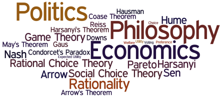
Since democracy, political equality, and majority rule are distinct ideas, each stands in need of separate justification.

B. Saunders. *Democracy, Political Equality, and Majority Rule*. Ethics, 2010.

Lottery Voting

Lottery voting: each person casts a vote for their favored option but, rather than the option with most votes automatically winning, a single vote is randomly selected and that one determines the outcome.

Lottery Voting



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- It is egalitarian, since all have an equal chance of being picked. It gives each voter an equal chance of being decisive, but voters do not have equal chances of getting their way—rather, the chance of each option winning is proportional to the number of votes it obtains.
- It is not majority rule, since the vote of someone in the minority may be picked.

Lottery Voting

This shows that democracy and political equality do not conceptually require majority rule.

(Saunders argues that there are no clearly decisive general reasons to prefer majority rule to lottery voting in all cases.)

What justifies majority rule?

1. Minority vs. Majority: If a minority could prevail over the majority, those who were in favor of a proposition would vote against it, or would abstain from voting in order to insure a majority to their side of the question. Also, there would be no inducement to discuss a question, if, by converting a person to our opinion, you did not strengthen your side when the votes came to be counted.

M. Risse. *Arguing for majority rule*. Journal of Political Philosophy 12 (1), pp. 41 - 64 (2004).

What justifies majority rule?

2. Respect: Majority rule is a good way of expressing respect for people in the circumstances of politics, that is, in circumstances in which in spite of remaining differences (even after deliberation) a common view needs to be found. Majority rule allows each person to remain faithful to their conviction, but still to accept that a group decision needs to be made.

M. Risse. *Arguing for majority rule*. Journal of Political Philosophy 12 (1), pp. 41 - 64 (2004).

What justifies majority rule?

3. Compromise: The “distance” between an individual’s ranking and the group ranking is a measure of her satisfaction with the group outcome. By minimizing the distance of individual rankings from the group ranking, majority rule maximizes overall satisfaction with the group choice.

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When there are only two options, can we argue that majority rule is the “best” procedure?

Setting aside the possibility of using lotteries, May’s Theorem is a proceduralist justification of majority rule showing that it is the unique procedure satisfying normative principles of group decision making.

K. May. *A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision*. Econometrica, Vol. 20 (1952).

May's Theorem: Details

Voters: $V = \{1, 2, 3, \ldots, n\}$ is the set of n voters.

Candidates: $X = \{a, b\}$ is set of candidates.

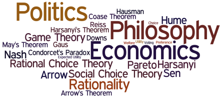
Suppose that voters can submit one of 3 rankings:

1. $a\ P\ b$: a is ranked above b ("vote for a")
2. $a\ I\ b$: a and b are tied ("vote for a and b")
3. $b\ P\ a$: b is ranked above a ("vote for b")

Note that $a\ I\ b$ and $b\ I\ a$ is the same ballot since indifference is symmetric.

Let $O(X)$ be the set of 3 rankings on X.

May's Theorem: Details



The set of **profiles** is $O(X)^V$, where a profile assigns to each voter one of the three rankings from $O(X)$.

Given a profile $\mathbf{P} \in O(X)^V$ and a voter $i \in V$, we write $\mathbf{P}_i$ for the ranking of voter i.

E.g., suppose that $V = \{1, 2, 3, 4\}$ and consider the profile

$$\mathbf{P} = (a\ P\ b, a\ I\ b, b\ P\ a, a\ P\ b)$$

Then, $\mathbf{P}_2$ is the ranking $a\ I\ b$ (voter 2 is indifferent between a and b).

May’s Theorem: Details

Social Choice Function: $F : O(X)^V \rightarrow \wp(X)$.

Where for all profiles $\mathbf{P}$ from $O(X)^V$, $F(\mathbf{P})$ is the set of winners.

We assume that for all profile $\mathbf{P}$, $F(\mathbf{P}) \neq \varnothing$ (so there is always at least one winner).

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Examples:

- Majority rule: The winner is the candidate with the most votes, otherwise the candidates are tied
- Quota rule: The winner is the candidate with more than $q\%$ of the vote (e.g., more than 2/3 of the vote), otherwise the candidates are tied.
- Unanimity rule: A candidate wins is *all voters* vote for that candidate, otherwise the candidates are tied.

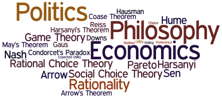
May's Theorem: Details

Social Choice Function: $F : O(X)^V \to \wp(X)$.

Examples:

- Minority rule: The winner is the candidate with the fewest votes, otherwise the candidates are tied.
- Majority rule with status quo: The winner is the candidate with the most votes, and if there is a tie candidate a wins.
- Candidate a always wins.
- The winner is whoever voter 1 voted for.
- The candidates are always tied.

May’s Theorem: Details



$$F_{Maj}(\mathbf{P}) = \begin{cases} \{a\} & \text{if more voters rank } a \text{ above } b \text{ than } b \text{ above } a \\ \{a, b\} & \text{if the same number of voters rank } a \text{ above } b \text{ as } b \text{ above } a \\ \{b\} & \text{if more voters rank } b \text{ above } a \text{ than } a \text{ above } b \end{cases}$$

May's Theorem: Details

$$F_{Maj}(\mathbf{P}) = \begin{cases} \{a\} & \text{if } Margin_{\mathbf{R}}(a,b) > 0 \\ \{a,b\} & \text{if } Margin_{\mathbf{R}}(a,b) = 0 \\ \{b\} & \text{if } Margin_{\mathbf{R}}(b,a) > 0 \end{cases}$$

Anonymity and Neutrality

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$\Longrightarrow$ in 2-candidate profiles, if the same number of voters rank $\boldsymbol{a}$ above $\boldsymbol{b}$ as $\boldsymbol{b}$ above $\boldsymbol{a}$, then $\boldsymbol{a} \in F(\mathbf{P})$ if, and only if, $\boldsymbol{b} \in F(\mathbf{P})$

($\boldsymbol{a}$ wins according to F if and only if $\boldsymbol{b}$ wins according to F).