PHPE 400
Individual and Group Decision Making

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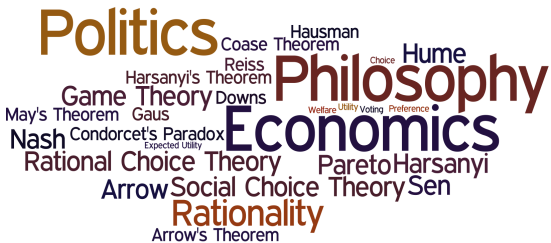
pacuit.org



Iterative Voting Methods

Instant Runoff Voting (Ranked Choice): Iteratively remove all candidates with the fewest number of voters who rank them first, until there is a candidate with a majority of first-place votes. If, at some stage of the removal process, all remaining candidates have the same number of voters who rank them first (so all candidates would be removed), then all remaining candidates are selected as winners.

Coombs: Iteratively remove all candidates with the most number of voters who rank them last, until there is a candidate with a majority of first-place votes. If, at some stage of the removal process, all remaining candidates have the same number of voters who rank them last (so all candidates would be removed), then all remaining candidates are selected as winners.

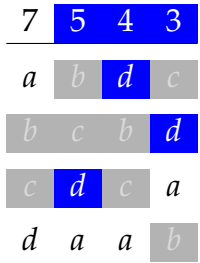
7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b

Instant Runoff winners

7	5	4	3
a	*b*	*d*	*c*
b	*c*	*b*	*d*
c	*d*	*c*	*a*
d	*a*	*a*	*b*

Instant Runoff winners

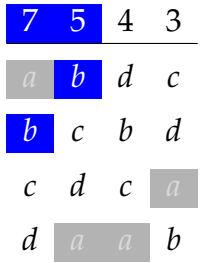


Instant Runoff winners d

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b

Instant Runoff winners d

Coombs winners



Instant Runoff winners	d
Coombs winners	b

2022 Alaska Special General Election

The Ranked Choice, also called Instant Runoff Voting (IRV), winner is Peltola.

- The write-ins are initially removed
- Begich is removed in the first round
- Palin loses to Peltola

2022 Alaska Special General Election

Round 1

Candidate	Votes	Percentage
Begich, Nick	53,810	28.53%
Palin, Sarah	58,973	31.27%
Peltola, Mary S.	75,799	40.19%
Continuing Ballots Total	**188,582**	
Blanks	3,412	
Exhausted	0	
Overvotes	295	
Remainder Points	0	
Non Transferable Total	**3,707**	

Begich, Nick is eliminated because the candidate had the least amount of votes.

2022 Alaska Special General Election

Elimination transfer for candidate Begich, Nick.

53810 ballots have been transferred in the following manner:

Transferred from	Transferred to	Ballots	Votes
Begich, Nick	Palin, Sarah	27053	27,053
Begich, Nick	Peltola, Mary S.	15467	15,467
Begich, Nick	Exhausted	11243	11,243
Begich, Nick	Overvotes	47	47

2022 Alaska Special General Election

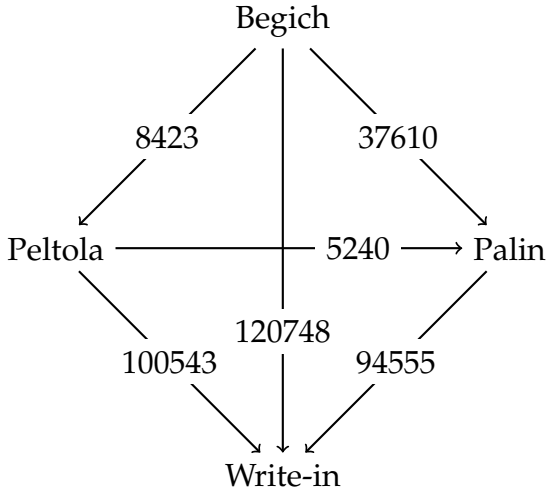
Round 2

Candidate	Votes	Percentage
Begich, Nick	0	0.00%
Palin, Sarah	86,026	48.52%
Peltola, Mary S.	91,266	51.48%
Continuing Ballots Total	**177,292**	
Blanks	3,412	
Exhausted	11,243	
Overvotes	342	
Remainder Points	0	
Non Transferable Total	**14,997**	

Palin, Sarah is eliminated because the candidate was not elected in the last round.

Problem 1: A majority of voters strictly prefer Begich to Peltola.

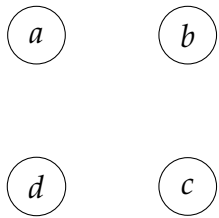
Problem 2: In fact, Begich is majority preferred to every other candidate, but is not elected.



IRV may not elect the Condorcet winner: The Condorcet winner is Begich, but Peltola was elected.

Recall Condorcet's Idea

3	5	7	6
a	a	b	c
b	c	d	b
c	b	c	d
d	d	a	a



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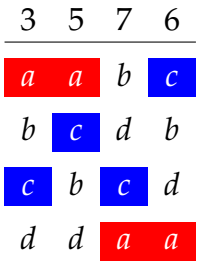
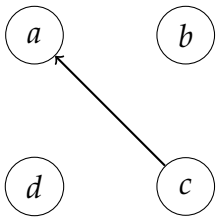
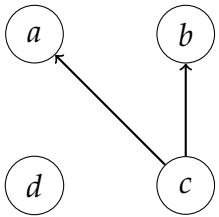




Recall Condorcet's Idea

3	5	7	6
a	*a*	*b*	*c*
b	*c*	*d*	*b*
c	*b*	*c*	*d*
d	*d*	*a*	*a*



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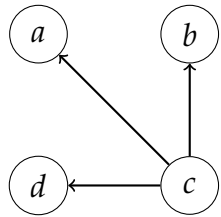

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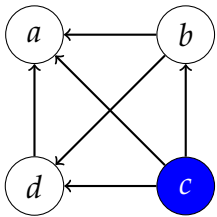

Condorcet Consistency

The **Condorcet winner** in a profile **P** is a candidate x such that for all other candidates y, $Margin_{\mathbf{P}}(x, y) > 0$.

A voting method is **Condorcet consistent**, if for all **P**, if x is a Condorcet winner in **P**, then x is the unique winner according to the voting method.

Borda, Plurality, Plurality with Runoff, Instant Runoff Voting, Coombs are **not** Condorcet consistent.

Can we find a voting method that is Condorcet consistent?

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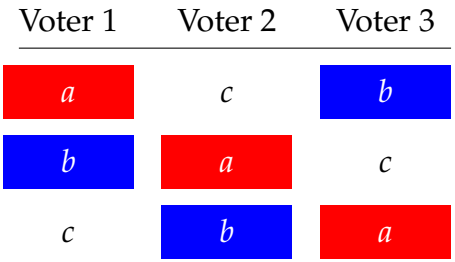
Can we find a voting method that is Condorcet consistent?

What about the method $F_{cond}(\mathbf{P}) = \{a\}$ where a is the Condorcet winner in $\mathbf{P}$?

The Problem

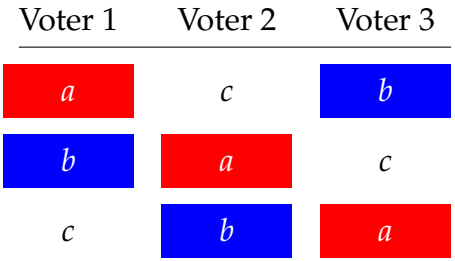
Voter 1	Voter 2	Voter 3
a	c	b
b	a	c
c	b	a

The Problem



- Does the group prefer a over b?

The Problem



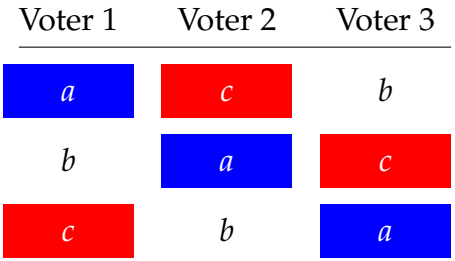
- Does the group prefer a over b? Yes

The Problem

Voter 1	Voter 2	Voter 3
a	c	b
b	a	c
c	b	a

- Does the group prefer a over b? Yes
- Does the group prefer b over c? Yes

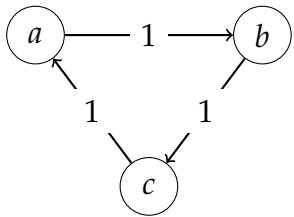
The Problem



- Does the group prefer a over b? Yes
- Does the group prefer b over c? Yes
- Does the group prefer a over c? No

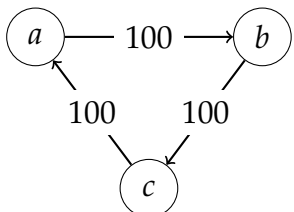
Majority Cycle Example

1	1	1
a	c	b
b	a	c
c	b	a



Majority Cycle Example

100	100	100
a	c	b
b	a	c
c	b	a



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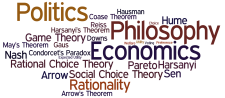
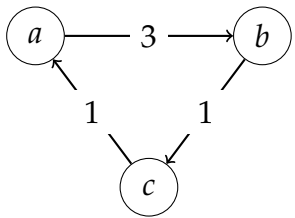
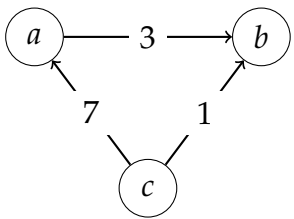


2	2	1
a	c	b
b	a	c
c	b	a



Not a Majority Cycle

1	5	3
a	c	b
b	a	c
c	b	a



Majority Cycles

A **majority cycle** is a list of candidates such that each has a positive margin over the next, and the last has a positive margin over the first.

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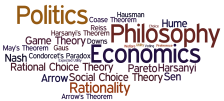

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- How *likely* is a majority cycle?

Majority Cycles - Examples

The 2007 Glasgow City Council election for Ward 5 (Govan): The election was run using Single-Transferable Vote to elect four candidates, but we can also imagine selecting a single winner based on these ballots.

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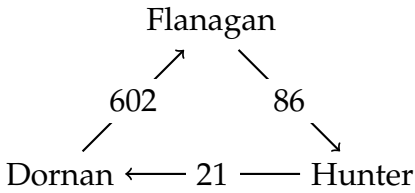
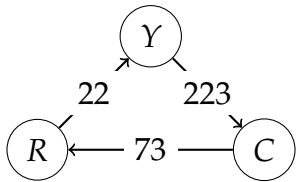


https://github.com/voting-tools/election-analysis/blob/main/glasgow_govan_2007.ipynb

Majority Cycles - Examples

The 2021 Minneapolis City Council Election (Ward 2):



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Condorcet consistent voting methods

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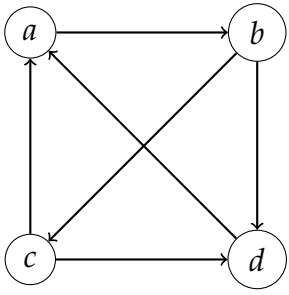
The Condorcet voting method Nanson was used in Marquette, Michigan, in the 1920s (Hoag and Hallett 1926, p. 491). To my knowledge, there are no cities using Condorcet consistent voting methods, but see the Condorcet Canada Initiative at `https://condorcet.ca`.

Copeland

Say that the **win-loss record** for a candidate x is the number of candidates that x is majority preferred to minus the number of candidates that is majority preferred to y.

Then, any candidate with the largest win-loss record is a Copeland winner.

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b



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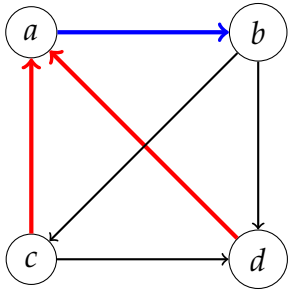
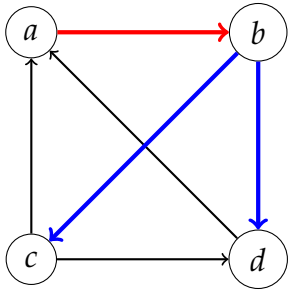


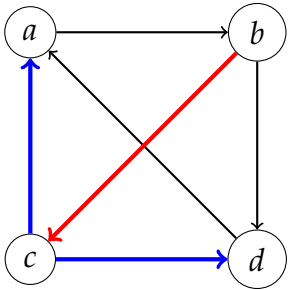
Win-loss record for a: $1 - 2 = -1$

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b



Win-loss record for a: $1 - 2 = -1$
Win-loss record for b: $2 - 1 = 1$

7	5	4	3
a	b	d	c
b	c	b	d
c	d	c	a
d	a	a	b



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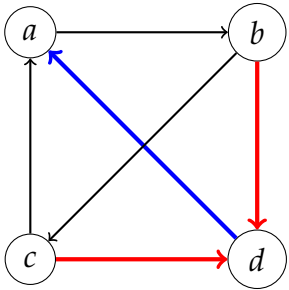
b	c	b	d
c	d	c	a
d	a	a	b



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Win-loss record for d: $1 - 2 = -1$

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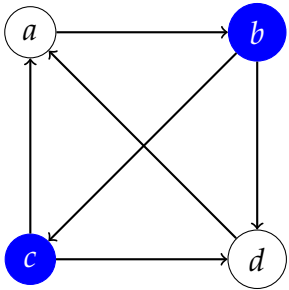
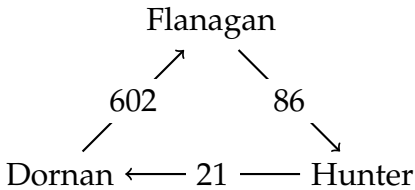


Win-loss record for a: $1 - 2 = -1$
Win-loss record for b: $2 - 1 = 1$
Win-loss record for c: $2 - 1 = 1$
Win-loss record for d: $1 - 2 = -1$
c and b are the Copeland winners.

2007 Glasgow City Council

The top three candidates were in a **majority cycle**:



All candidates are tied according to Copeland (each candidate's win-loss record is 0).

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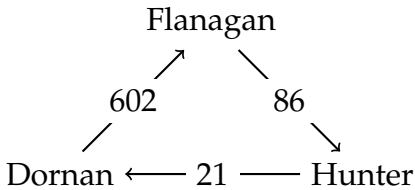

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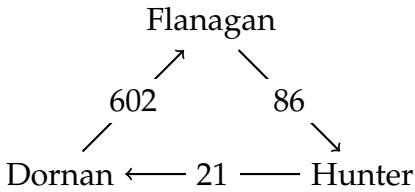

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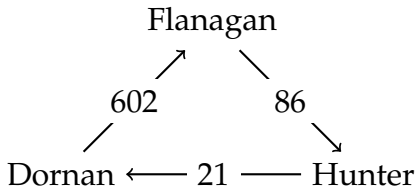
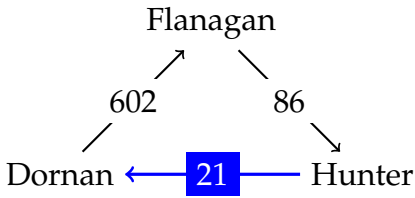
It's Dornan.

Minimax

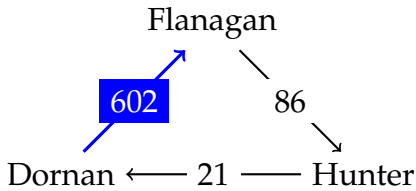
Say that the head-to-head loss of x vs. y is the margin of y over x: the number of voters that rank y above x minus the number of voters that rank x above y.

Find the largest head-to-head loss for each candidate. Any candidate with the smallest such loss is a Minimax winner.


Flanagan
602
86
Dornan
21
Hunter



The largest head-to-head loss of Dornan is 21



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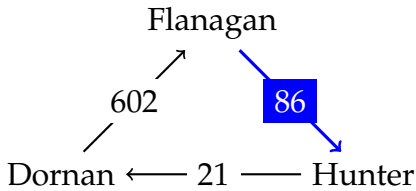

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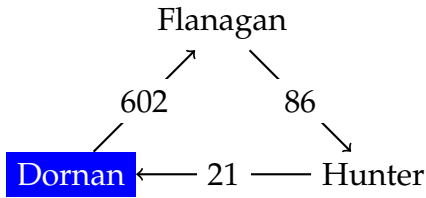
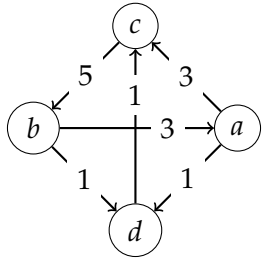
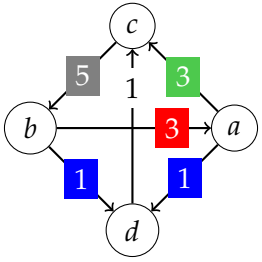
The largest head-to-head loss of Flanagan is 602
The largest head-to-head loss of Hunter is 86

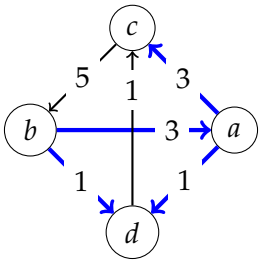


The largest head-to-head loss of Dornan is 21
The largest head-to-head loss of Flanagan is 602
The largest head-to-head loss of Hunter is 86
Dornan is the Minimax winner.


c
5
1
3
b
3
a
1
1
d



d is the Minimax winner.



d is the Minimax winner.
a and b are the Copeland winners.

Split Cycle

1. In each majority cycle, identify the wins with the smallest margin in that cycle.

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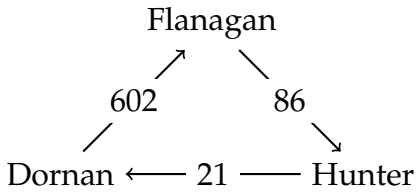

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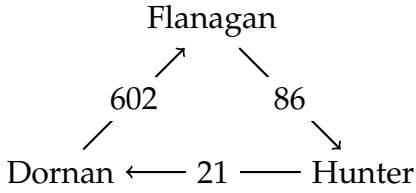
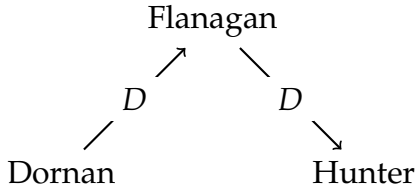
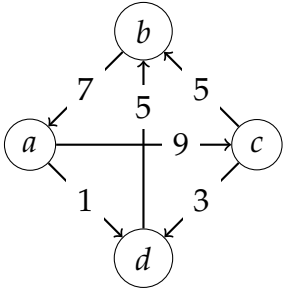




Example

Suppose an election produces the following majority margin graph (i.e., there are 7 more voters who ranked ***b*** above ***a*** than who ranked ***a*** above ***b***, etc.):



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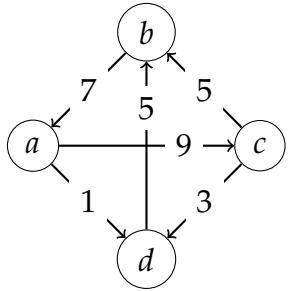
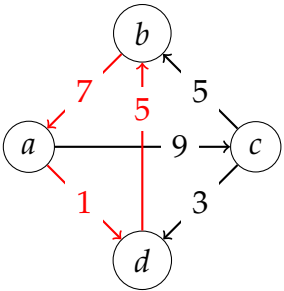
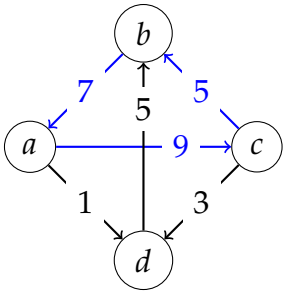
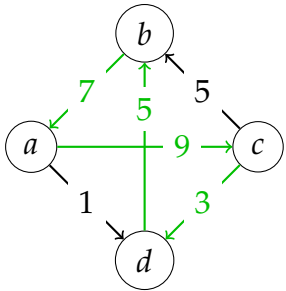


Our first step is to identify the cycles…

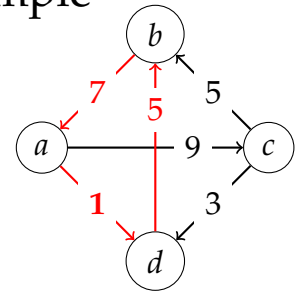
Example

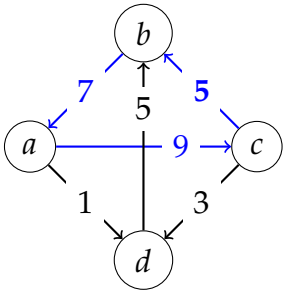


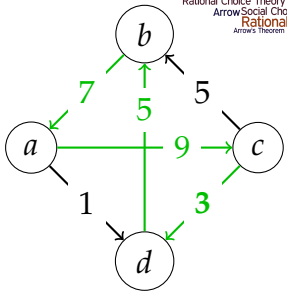




Example

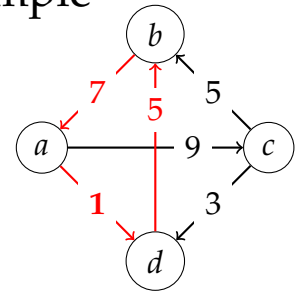


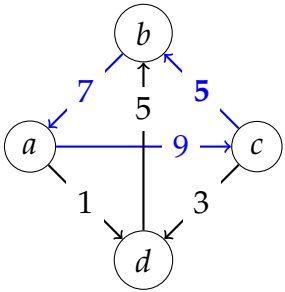


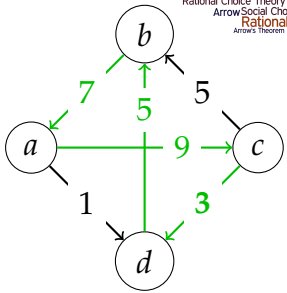


Next find the smallest margin in each cycle.

Example

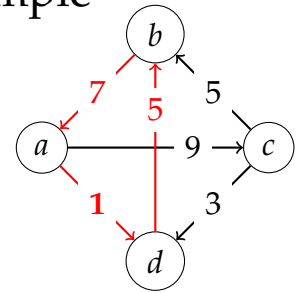


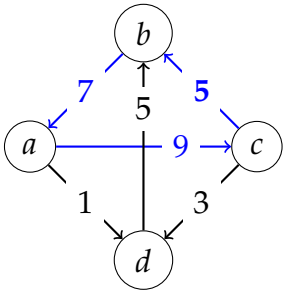


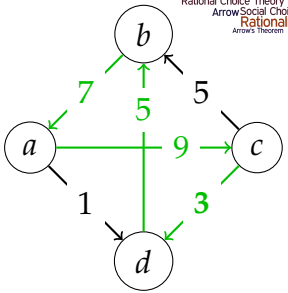


Next find the smallest margin in each cycle.
These edges cannot be defeats.

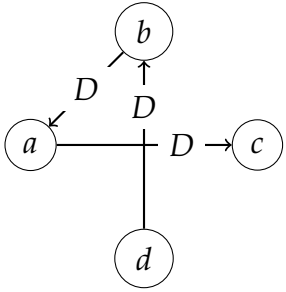
Example







Next find the smallest margin in each cycle.
These edges cannot be defeats.
But all other edges are defeats.
d is the Split Cycle winner.



Voting Method Tutorial

https://voting-tutorial.streamlit.app/

Which Voting Method is Best?

A 2004 letter to the Washington Post sent by a local organizer of the Green Party, as quoted by Miller (2019, p. 119):

> *[Electoral engineering] isn't rocket science. Why is it that we can put a man on the moon but can't come up with a way to elect our president that allows voters to vote for their favorite candidate, allows multiple candidates to run and present their issues and…[makes] the 'spoiler' problem…go away?*