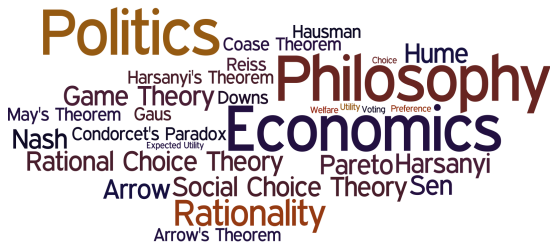


# PHPE 400

## Individual and Group Decision Making

Eric Pacuit  
University of Maryland  
[pacuit.org](http://pacuit.org)



Voters    Rankings

1

*a b c d*

2

*b a d c*

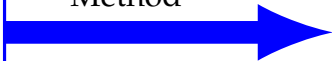
3

*b d a c*

4

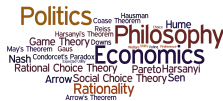
*d c a b*

Voting  
Method



Winning Set

# There are many different voting methods



Plurality, Borda Count, Antiplurality/Veto; Coombs; (Strict/Weak) Nanson; Baldwin, Plurality with Runoff; Rank Choice/Single Transferable Vote (STV)/Hare; Copeland $_{\alpha}$ ; Bucklin; Minimax; Beat Path; Split Cycle; Stable Voting; Ranked Pairs; GETCHA; GOCHA; Kemeny; Dodgson Method; Young's Method; Approval Voting; Majority Judgment; Cumulative Voting; Range/Score Voting; . . .

# Electoral Reform



## New York City Voters Just Adopted Ranked-Choice Voting in Elections. Here's How It Works



Public schools represent the voting precincts for most of a public school polling location in New York City. A long

## The Rules of the Game: A New Electoral System

By Matt Mahler and Robert Post



The New York Times

Opinion

Column

## How Majority Rule Might Have Stopped Donald Trump

By John M. Heilbrunn

July 20, 2016

1 2 3 4 5 6 7 8 9 10

Cambridge, Mass. — DEMOCRATIC REP has won legislative elections in 10 states in this — far more than any of his opponents. Yet in the first 27 states he won, several of the other major candidates might well have been in a serious race.

There is no contradiction here. In the early months, Mr. Trump attracted less than 10 percent of the vote in California but got only 23 percent of the votes in that state. But he had more than one opponent every time, so that the vote Trump cast was split. That implies he could not have been declared in most (given the experience there on local elections) had the appropriate conditional precedent of his leading rivals. In such a scenario, he might have been one of several leading before the runoff vote his plurality vote would have lost outright majority vote — in New York, last month.

American politicians are not the only recent elections to produce winners lacking the support of a majority of voters. In both, the Republican leaders they received only 23 percent of the vote in the last general election, but got a majority of parliamentary seats. Given including political elites, their vote share was no more than 20 percent. The B.F.P. is a state-wide party with a 10-day history.

POLITICO

NEWSLETTER



ELECTIONS

## New York's 'head-swirling' mistake puts harsh spotlight on ranked-choice voting

The weeks without a winner in the hard-fought primary for control of the governor's office have begun to resemble their counterparts.





# Electoral Reform



**New York City Voters Just Adopted Ranked-Choice Voting in Elections. Here's How It Works**



Public ballot stations for voting from the start of a public school polling location in New York on May 3, 2019.

**The Rules of the Game: A New Electoral System**

By Matt and Robert Post



Opinion

**How Majority Rule Might Have Stopped Donald Trump**

By John Madeiras and Ben

May 10, 2017

Cambridge, Mass. — DEMOCRATIC REP has won legislative victories in 10 states so far — far more than any of the opposition. Yet in the first 27 states he won, several of the other major candidates might well have beaten him in a one-on-one runoff.

There is no constitutional bar, in the early states, Mr. Trump entered less than 50 percent of the vote. Mr. Clinton got only 23 percent, a majority of voters opposed him. But he lost even close one-on-one runoffs, so that the one Trump overtook right. That implies he could not have been defeated in most (given his opponent there no hard evidence that the opposition considered runoff over of his leading rivals. In such a scenario, he might have been out of contention before he could vote his plurality. A runoff would be the one might easily win — in New York, last month.

American politicians are not the only recent elections to produce winners lacking the support of a majority of voters. In both, the Republican Senate Party received only 23 percent of the vote in the last general election, but got a majority of parliamentary seats. Given including political allies, their vote share was no more than 30 percent. The R.F.F. is a state vote carry with a 30-state majority.

POLITICO



**New York's 'head-swirling' mistake puts harsh spotlight on ranked-choice voting**

This week without a winner in the hard-fought mayoral primary has ranked-choice backers desperate to maintain their momentum.



- ▶ Ranked-Choice Voting: FairVote (<http://www.fairvote.org>)
- ▶ Approval Vote: Center for Election Science (<https://www.electology.org>)

# Electoral Reform



New York City Voters Just Adopted Ranked-Choice Voting in Elections. Here's How It Works



Public ballot stations represent the voting process for voters at a public school polling location in New York City on May 3, 2019.

The Rules of the Game: A New Electoral System



Opinion

How Majority Rule Might Have Stopped Donald Trump

By John M. Heilbrunn

July 26, 2019

Cambridge, Mass. — DEMOCRATIC REP has won legislative elections in 10 states in three — far more than any of his opponents. Yet in the first 17 months he was governor, the other major candidates might well have been known as a one-term governor.

There is no contradiction here. In the early months, Mr. Trump attracted less than 50 percent of the vote. Mr. DeSantis fought only 20 percent of a runoff against him. But he found more than one opponent every time, so that like the man Trump overtook right, that implies he could not have been defeated in most (given the experience there on local ballots) that the opposition considered a worst case of his handling. In such a scenario, he might have been out of commission before he could take his victory. A victory would be that might equally well — in New York, last month.

American politicians are not the only recent elections to produce winners lacking the support of a majority of voters. In both, the Republican leaders they received only 20 percent of the vote in the last general election, but got a majority of parliamentary seats. Given including political allies, their vote share was no more than 30 percent. The U.K. U.K. is a state-wide party with a 30-day history.

POLITICO



New York's 'head-swirling' mistake puts harsh spotlight on ranked-choice voting

This week without a winner in the 2020 election, many primary and caucus states are expected to maintain their momentum.



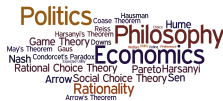
- ▶ Ranked-Choice Voting: FairVote (<http://www.fairvote.org>)
- ▶ Approval Vote: Center for Election Science (<https://www.electology.org>)
- ▶ Open primaries?
- ▶ Electoral college?
- ▶ How do you draw voting districts?

# Choosing how to choose



**Pragmatic considerations:** Is the procedure easy to use? Is it legal? The importance of *ease of use* should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.

# Choosing how to choose



**Pragmatic considerations:** Is the procedure easy to use? Is it legal? The importance of *ease of use* should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.

**Information required from the voters:** What type of information do the ballots convey? I.e., Choosing a single alternative, linearly rank all the candidates, report something about the “intensity” of preference.

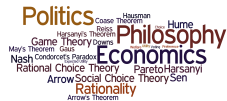


# Notation



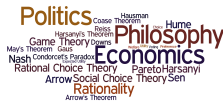
- ▶  $V$  is a finite set of voters (assume that  $V = \{1, 2, 3, \dots, n\}$ )
- ▶  $X$  is a (typically finite) set of alternatives, or candidates
- ▶ An election **profile** is a record of the **ballot** submitted by each voter, where a ballot can be any of the following:
  - ▶ A selected candidate
  - ▶ A ranking of the candidates
  - ▶ Scores/grades assigned to each candidate

# Rankings



SAND COUNTY						
MAYOR 市長	1 1st Choice 第一選擇	2 2nd Choice 第二選擇	3 3rd Choice 第三選擇	4 4th Choice 第四選擇	5 5th Choice 第五選擇	6 6th Choice 第六選擇
ELLEN LEE ZHOU / 李麗晨 Behavioral Health Clinician 行為健康臨床治療師	● <sup>1</sup>	● <sup>2</sup>	● <sup>3</sup>	● <sup>4</sup>	● <sup>5</sup>	● <sup>6</sup>
LONDON N. BREED / 倫敦·布里德 Mayor of San Francisco 三藩市市長	● <sup>1</sup>	● <sup>2</sup>	● <sup>3</sup>	● <sup>4</sup>	● <sup>5</sup>	● <sup>6</sup>
JOEL VENTRESCA / 喬爾·范崔斯卡 Retired Airport Analyst 退休機場分析師	● <sup>1</sup>	● <sup>2</sup>	● <sup>3</sup>	● <sup>4</sup>	● <sup>5</sup>	● <sup>6</sup>
WILMA PANG / 彭德慧 Retired Music Professor 退休音樂教授	● <sup>1</sup>	● <sup>2</sup>	● <sup>3</sup>	● <sup>4</sup>	● <sup>5</sup>	● <sup>6</sup>
ROBERT L. JORDAN, JR. / 小羅伯特·L·喬丹 Preacher 傳教士	● <sup>1</sup>	● <sup>2</sup>	● <sup>3</sup>	● <sup>4</sup>	● <sup>5</sup>	● <sup>6</sup>
PAUL YBARRA ROBERTSON / 保羅·伊巴拉·羅伯森 Small Business Owner 小企業業主	● <sup>1</sup>	● <sup>2</sup>	● <sup>3</sup>	● <sup>4</sup>	● <sup>5</sup>	● <sup>6</sup>
	● <sup>1</sup>	● <sup>2</sup>	● <sup>3</sup>	● <sup>4</sup>	● <sup>5</sup>	● <sup>6</sup>

# Rankings



Let  $X$  be a set of candidates and  $V$  a set of voters.

A **ranking** of  $X$  is a strict linear order  $P$  on  $X$ : a relation  $P \subseteq X \times X$  satisfying the following conditions for all  $x, y, z \in X$ :

*asymmetry*: if  $x P y$  then *not*  $y P x$ ;

*transitivity*: if  $x P y$  and  $y P z$ , then  $x P z$ ;

*weak completeness*: if  $x \neq y$ , then  $x P y$  or  $y P x$ .

Let  $L(X)$  be the set of all strict linear orders on  $X$ .



# Profiles



A **profile** for  $X$  is a function  $\mathbf{P}$  assigning to  $i \in V$  a linear order  $\mathbf{P}_i$  on  $X$ .

# Profiles



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So,  $a \mathbf{P}_i b$  means that voter  $i$  ranks  $a$  above  $b$ , or that  $i$  strictly prefers candidate  $a$  to  $b$ .

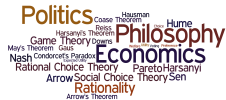
For instance,

Example: let  $V = \{v_1, v_2, v_3, v_4\}$  and  $X = \{a, b, c, d\}$  and consider the following profile  $\mathbf{P}$ ,

$v_1$	$v_2$	$v_3$	$v_4$
$a$	$a$	$b$	$c$
$b$	$c$	$a$	$b$
$c$	$b$	$c$	$a$

E.g.,  $a \mathbf{P}_{v_2} c$ ,  $b \mathbf{P}_{v_4} a$ ,  $a \mathbf{P}_{v_1} b$ , ...

# Anonymous Profiles



An **anonymous profile** is a function  $\rho : L(X) \rightarrow \mathbb{N}$ , where  $L(X)$  is the set of rankings of  $X$ .

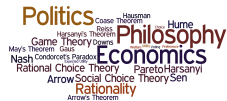
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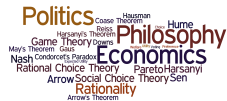
2	5	3	5
$a$	$a$	$b$	$c$
$b$	$c$	$a$	$b$
$c$	$b$	$c$	$a$

# (Linear) Profiles



$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$	$v_7$	$v_8$	$v_9$	$v_{10}$	$v_{11}$	$v_{12}$	$v_{13}$	$v_{14}$	$v_{15}$
$b$	$b$	$b$	$b$	$b$	$b$	$b$	$a$	$a$	$a$	$a$	$a$	$a$	$a$	$a$
$c$	$c$	$c$	$c$	$c$	$c$	$c$	$c$	$c$	$c$	$c$	$c$	$b$	$b$	$b$
$a$	$a$	$a$	$a$	$a$	$a$	$a$	$b$	$b$	$b$	$b$	$b$	$c$	$c$	$c$

# (Linear) Anonymous Profile



$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$	$v_7$	$v_8$	$v_9$	$v_{10}$	$v_{11}$	$v_{12}$	$v_{13}$	$v_{14}$	$v_{15}$
$b$	$b$	$b$	$b$	$b$	$b$	$b$	$a$	$a$	$a$	$a$	$a$	$a$	$a$	$a$
$c$	$c$	$c$	$c$	$c$	$c$	$c$	$c$	$c$	$c$	$c$	$c$	$b$	$b$	$b$
$a$	$a$	$a$	$a$	$a$	$a$	$a$	$b$	$b$	$b$	$b$	$b$	$c$	$c$	$c$

7	5	3
$b$	$a$	$a$
$c$	$c$	$b$
$a$	$b$	$c$



# Majoritarianism



When there are only **two** candidates  $a$  and  $b$ , then all (reasonable) voting methods give the same results:



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**Majority Rule:**  $a$  is the winner if more than  $1/2$  of the voters rank  $a$  above  $b$ ,  $b$  is the winner if more than  $1/2$  of votes rank  $b$  above  $a$ , otherwise  $a$  and  $b$  are tied.

# Majoritarianism

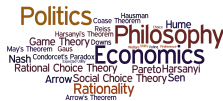


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Yes. We will look at two arguments: A procedural justification and an epistemic justification.

# Majoritarianism



What about when there are *more than* two candidates, can we still argue that majority rule is the “best” procedure?

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Results are more mixed: Consider our previous definition of majority rule....

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Results are more mixed: Consider our previous definition of majority rule....we only defined it between two options! Can we generalize for  $|X| > 2$ ?

# Majoritarianism

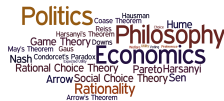


What about when there are *more than* two candidates, can we still argue that majority rule is the “best” procedure?

Results are more mixed: Consider our previous definition of majority rule....we only defined it between two options! Can we generalize for  $|X| > 2$ ?

The problem is that with more than 2 candidates, there may not be any candidate that is ranked first by more than half of the voters.

# Important Distinction



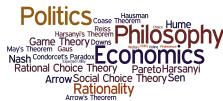
1	1	1
<i>a</i>	<i>a</i>	<i>d</i>
<i>b</i>	<i>c</i>	<i>a</i>
<i>c</i>	<i>d</i>	<i>b</i>
<i>d</i>	<i>b</i>	<i>c</i>

Do all of the voters rank *a* and *b* in the same way?

Do all of the voters rank *a* and *b* in the same *position*?



# Important Distinction



1	1	1
<i>a</i>	<i>a</i>	<i>d</i>
<i>b</i>	<i>c</i>	<i>a</i>
<i>c</i>	<i>d</i>	<i>b</i>
<i>d</i>	<i>b</i>	<i>c</i>

Do all of the voters rank *a* and *b* in the same way?

Yes: All of the voters rank *a* above *b*.

Do all of the voters rank *a* and *b* in the same *position*?

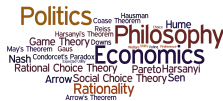
No: The first group ranks *a* in first-place and *b* in second-place, the second group ranks *a* in first-place and *b* is last place, and the third group ranks *a* is second-place and *b* in third-place.

# Positional scoring rules



A **scoring rule** each voter submits a ranking of the candidates. Based on the ranking, each voter assigns a *score* to each candidate. The candidate's overall score is the sum of the scores assigned to the candidate by each voter. Then, the candidate(s) with the greatest overall score is(are) the winner(s).

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- ▶ **Plurality:** Each voter assigns a score of 1 to the candidate ranked in first place and 0 to all other candidates.
- ▶ **Borda:** If there are  $n$  candidates, then each voter assigns a score of  $n - 1$  to the candidate in first place,  $n - 2$  to the candidate in 2nd place,  $\dots$ , and 0 to the candidate in last place.

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

	7	5	4	3
1	<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
0	<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
0	<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
0	<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Plurality winner(s): *a*

Plurality score of <i>a</i> :	$1 * 7$	+	$0 * 0$	+	$0 * 3$	+	$0 * 9$	=	7
Plurality score of <i>b</i> :	$1 * 5$	+	$0 * 11$	+	$0 * 0$	+	$0 * 3$	=	5
Plurality score of <i>c</i> :	$1 * 4$	+	$0 * 5$	+	$0 * 11$	+	$0 * 0$	=	4
Plurality score of <i>d</i> :	$1 * 3$	+	$0 * 3$	+	$0 * 5$	+	$0 * 7$	=	3

	7	5	4	3
3	<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
2	<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
1	<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
0	<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Borda winner(s): *b*

Borda score of <i>a</i> :	$3 * 7$	+	$2 * 0$	+	$1 * 3$	+	$0 * 9$	=	24
Borda score of <i>b</i> :	$3 * 5$	+	$2 * 11$	+	$1 * 0$	+	$0 * 3$	=	37
Borda score of <i>c</i> :	$3 * 4$	+	$2 * 5$	+	$1 * 11$	+	$0 * 0$	=	33
Borda score of <i>d</i> :	$3 * 3$	+	$2 * 3$	+	$1 * 5$	+	$0 * 7$	=	20

1	2	2
<hr/>		
$x$	$y$	$y$
$y$	$x$	$x$

Who are the Borda winners?  $y$

1	2	2
$x$	$y$	$y$
$a_1$	$x$	$x$
$a_2$	$a_1$	$a_1$
$a_3$	$a_2$	$a_2$
$y$	$a_3$	$a_3$

Who are the Borda winners?



	1	2	2
	$x$	$y$	$y$
$a_1$	$x$	$x$	$x$
$a_2$	$a_1$	$a_1$	$a_1$
$a_3$	$a_2$	$a_2$	$a_2$
$y$	$a_3$	$a_3$	$a_3$

Who are the Borda winners?  $x$  and  $y$

1	2	2
$x$	$y$	$y$
$a_1$	$x$	$x$
$a_2$	$a_1$	$a_1$
$a_3$	$a_2$	$a_2$
$a_4$	$a_3$	$a_3$
$y$	$a_4$	$a_4$

Who are the Borda winners?

1	2	2
$x$	$y$	$y$
$a_1$	$x$	$x$
$a_2$	$a_1$	$a_1$
$a_3$	$a_2$	$a_2$
$a_4$	$a_3$	$a_3$
$y$	$a_4$	$a_4$

Who are the Borda winners?  $x$ ,  
**but a majority of voters prefer  $y$  over  $x$ .**

When there is no majority winner, can we find the candidate(s) that are “closest” to the majority winner?

Let's start with an example involving the voting method known as **“Ranked Choice Voting,” “Instant Runoff,”** or **“Hare System.”**

This is widely used in Australia and is promoted in the U.S. by FairVote.org and the anti-corruption campaign RepresentUs.

**FairVote**  
FOR A MORE PERFECT UNION



# Instant Runoff (aka Ranked Choice)



Iteratively remove all candidates with the fewest number of voters who rank them first, until there is a candidate with a majority of first-place votes. If, at some stage of the removal process, all remaining candidates have the same number of voters who rank them first (so all candidates would be removed), then all remaining candidates are selected as winners.

Iteratively remove all candidates with the most number of voters who rank them last, until there is a candidate with a majority of first-place votes. If, at some stage of the removal process, all remaining candidates have the same number of voters who rank them last (so all candidates would be removed), then all remaining candidates are selected as winners.

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>



7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Instant Runoff winners

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Instant Runoff winners

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Instant Runoff winners  $d$

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Instant Runoff winners *d*

Coombs winners

7	5	4	3
<i>a</i>	<i>b</i>	<i>d</i>	<i>c</i>
<i>b</i>	<i>c</i>	<i>b</i>	<i>d</i>
<i>c</i>	<i>d</i>	<i>c</i>	<i>a</i>
<i>d</i>	<i>a</i>	<i>a</i>	<i>b</i>

Instant Runoff winners *d*

Coombs winners *b*