PHPE 400
Individual and Group Decision Making

Eric Pacuit
University of Maryland
pacuit.org

Rational choice

A decision maker chooses rationally if her preferences are rational and there is nothing available that the decision maker prefers to what she chooses.

Rational choice

A decision maker chooses rationally if her preferences are rational and there is nothing available that the decision maker prefers to what she chooses.

Mathematically describing preferences

notes.phpe400.info/mathematical-preliminaries/sets.html

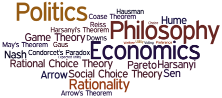
notes.phpe400.info/mathematical-preliminaries/relations.html

Answer the mathematical notation quiz on Tophat before your discussion section on Friday (the answers will be discussed during sections): `https://app.tophat.com/e/384276/content/1117900::f6a2a05b-dd5c-44fd-9297-fd322cfab11a?open_fullscreen=true`

Mathematical background: Relations

Suppose that X is a set. A **relation** on X is a set of **ordered pairs** from X: $R \subseteq X \times X$.

Mathematical background: Relations



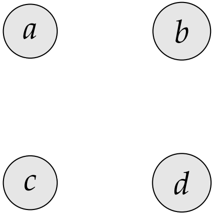
Suppose that X is a set. A **relation** on X is a set of **ordered pairs** from X: $R \subseteq X \times X$.

E.g., $X = \{a, b, c, d\}$, $R = \{(a, a), (b, a), (c, d), (a, c), (d, d)\}$

Mathematical background: Relations

Suppose that X is a set. A **relation** on X is a set of **ordered pairs** from X: $R \subseteq X \times X$.

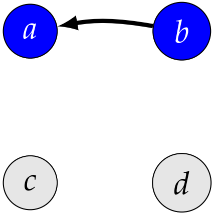
E.g., $X = \{a, b, c, d\}$, $R = \{(a, a), (b, a), (c, d), (a, c), (d, d)\}$



Mathematical background: Relations

Suppose that X is a set. A **relation** on X is a set of **ordered pairs** from X: $R \subseteq X \times X$.

E.g., $X = \{a, b, c, d\}$, $R = \{(a, a), (b, a), (c, d), (a, c), (d, d)\}$

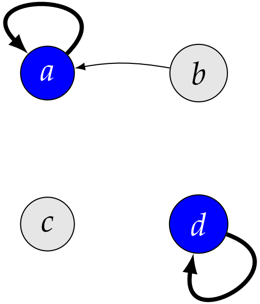


$b \; R \; a$

Mathematical background: Relations

Suppose that X is a set. A **relation** on X is a set of **ordered pairs** from X: $R \subseteq X \times X$.

E.g., $X = \{a, b, c, d\}$, $R = \{(a, a), (b, a), (c, d), (a, c), (d, d)\}$



$a \; R \; a$

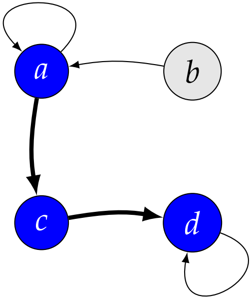
$b \; R \; a$

$d \; R \; d$

Mathematical background: Relations

Suppose that X is a set. A **relation** on X is a set of **ordered pairs** from X: $R \subseteq X \times X$.

Example: $X = \{a, b, c, d\}$, $R = \{(a, a), (b, a), (c, d), (a, c), (d, d)\}$



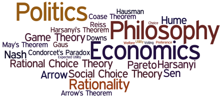
$a \; R \; a$
$b \; R \; a$
$c \; R \; d$
$a \; R \; c$
$d \; R \; d$

Representing Preferences

Let X be a set of outcomes. A decision maker's *preference* over X is represented by *relations* on X:

- $P \subseteq X \times X$ where $a\ P\ b$ means that the decision maker *strictly prefers* a to b.

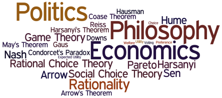
Representing Preferences



Let X be a set of outcomes. A decision maker's *preference* over X is represented by *relations* on X:

- $P \subseteq X \times X$ where $a \, P \, b$ means that the decision maker *strictly prefers* a to b.
- $I \subseteq X \times X$ where $a \, I \, b$ means that the decision maker is *indifferent* between a and b.

Representing Preferences



Let X be a set of outcomes. A decision maker's *preference* over X is represented by *relations* on X:

- $P \subseteq X \times X$ where $a\ P\ b$ means that the decision maker *strictly prefers* a to b.
- $I \subseteq X \times X$ where $a\ I\ b$ means that the decision maker is *indifferent* between a and b.
- $N \subseteq X \times X$ where $a\ N\ b$ means that the decision maker *cannot compare* a and b.

Strict Preference, I

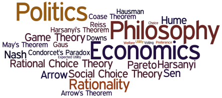
A decision maker's strict preference over a set X is represented as a relation $P \subseteq X \times X$.

Strict Preference, I

A decision maker's strict preference over a set X is represented as a relation $P \subseteq X \times X$.

The underlying idea is that if P represents the decision maker's strict preference and $x \; P \; y$ (i.e., the decision maker strictly prefers x to y), then the decision maker would pay some non-zero amount money to trade y for x.

Strict Preference, I



A decision maker's strict preference over a set X is represented as a relation $P \subseteq X \times X$.

The underlying idea is that if P represents the decision maker's strict preference and $x \; P \; y$ (i.e., the decision maker strictly prefers x to y), then the decision maker would pay some non-zero amount money to trade y for x.

P is asymmetric: for all $x, y \in X$, if $x \; P \; y$, then it is not the case that $y \; P \; x$ (written not-$y \; P \; x$).

Indifference/Incommensurable

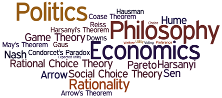
Suppose that P is an asymmetric relation on X (interpreted as a decision maker's strict preference). Suppose that $x, y \in X$ with not-$x\ P\ y$ and not-$y\ P\ x$.

Indifference/Incommensurable

Suppose that P is an asymmetric relation on X (interpreted as a decision maker's strict preference). Suppose that $x, y \in X$ with not-$x \; P \; y$ and not-$y \; P \; x$. There are two reasons why this might hold:

1. The decision maker is *indifferent* between x and y.
   In this case, we write $x \; I \; y$.
2. The decision maker *cannot compare* x and y.
   In this case, we write $x \; N \; y$.

Preferences



There are four distinct ways a decision maker can compare x and y:

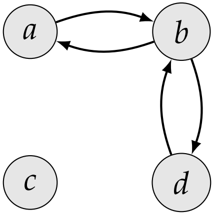
1. $x\ P\ y$: the decision maker *strictly prefers* x to y.
2. $y\ P\ x$: the decision maker *strictly prefers* y to x.
3. $x\ I\ y$: the decision maker is *indifferent* between x and y.
4. $x\ N\ y$: the decision maker *cannot compare* x and y.

Symmetric/Asymmetric Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Symmetric relation: for all $x, y \in X$, if $x\ R\ y$, then $y\ R\ x$

Asymmetric relation: for all $x, y \in X$, if $x\ R\ y$, then not-$y\ R\ x$



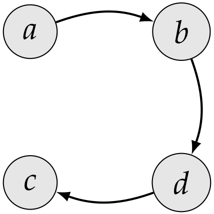
symmetric but not asymmetric

Symmetric/Asymmetric Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Symmetric relation: for all $x, y \in X$, if $x \mathrel{R} y$, then $y \mathrel{R} x$

Asymmetric relation: for all $x, y \in X$, if $x \mathrel{R} y$, then not-$y \mathrel{R} x$



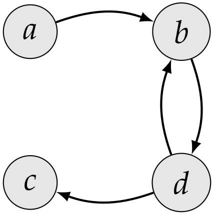
asymmetric but not symmetric

Symmetric/Asymmetric Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

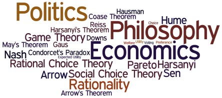
Symmetric relation: for all $x, y \in X$, if $x\ R\ y$, then $y\ R\ x$

Asymmetric relation: for all $x, y \in X$, if $x\ R\ y$, then not-$y\ R\ x$



not symmetric and not asymmetric

Symmetric/Asymmetric Relations



Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Symmetric relation: for all $x, y \in X$, if $x \mathrel{R} y$, then $y \mathrel{R} x$

Asymmetric relation: for all $x, y \in X$, if $x \mathrel{R} y$, then not-$y \mathrel{R} x$

Irreflexive relation: for all $x \in X$, if not-$x \mathrel{R} x$

Reflexive Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

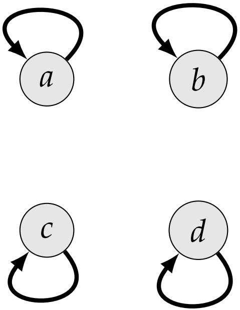
Reflexive relation: for all $x \in X$, $x \, R \, x$

Reflexive Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Reflexive relation: for all $x \in X$, $x\ R\ x$

E.g., $X = \{a, b, c, d\}$



Preferences - Minimal Constraints

A decision maker's preferences on X is represented by three relations $P \subseteq X \times X$, $I \subseteq X \times X$ and $N \subseteq X \times X$ satisfying the following minimal constraints:

1. For all $x, y \in X$, exactly one of $x\ P\ y$, $y\ P\ x$, $x\ I\ y$ and $x\ N\ y$ is true.
2. P is asymmetric
3. I is reflexive and symmetric.
4. N is symmetric.

A decision maker’s preferences are **rational** when they are transitive and complete.

A decision maker's preferences are **rational** when they are transitive and complete.

Transitive Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

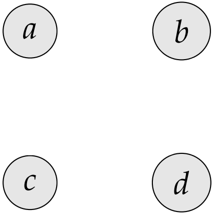
Transitive relation: for all $x, y, z \in X$, if $x \; R \; y$ and $y \; R \; z$, then $x \; R \; z$

Transitive Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Transitive relation: for all $x, y, z \in X$, if $x \; R \; y$ and $y \; R \; z$, then $x \; R \; z$

E.g., $X = \{a, b, c, d\}$

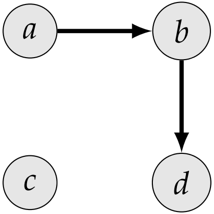


Transitive Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Transitive relation: for all $x, y, z \in X$, if $x\ R\ y$ and $y\ R\ z$, then $x\ R\ z$

E.g., $X = \{a, b, c, d\}$

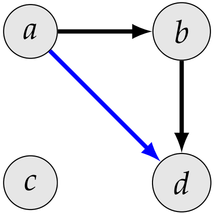


Transitive Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Transitive relation: for all $x, y, z \in X$, if $x \; R \; y$ and $y \; R \; z$, then $x \; R \; z$

E.g., $X = \{a, b, c, d\}$

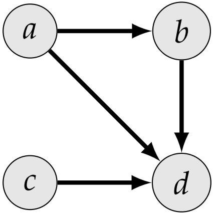


Transitive Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Transitive relation: for all $x, y, z \in X$, if $x \; R \; y$ and $y \; R \; z$, then $x \; R \; z$

E.g., $X = \{a, b, c, d\}$

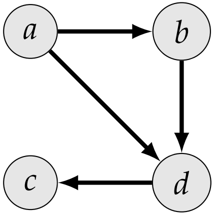


Transitive Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Transitive relation: for all $x, y, z \in X$, if $x\ R\ y$ and $y\ R\ z$, then $x\ R\ z$

E.g., $X = \{a, b, c, d\}$

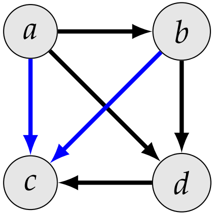


Transitive Relations

Suppose that X is a set and $R \subseteq X \times X$ is a relation.

Transitive relation: for all $x, y, z \in X$, if $x\ R\ y$ and $y\ R\ z$, then $x\ R\ z$

E.g., $X = \{a, b, c, d\}$



Transitivity

Strict preference is transitive: for all x, y, z if $x \; P \; y$ and $y \; P \; z$ then $x \; P \; z$

Transitivity

Strict preference is transitive: for all x, y, z if $x\ P\ y$ and $y\ P\ z$ then $x\ P\ z$

? Indifference is transitive: for all x, y, z if $x\ I\ y$ and $y\ I\ z$ then $x\ I\ z$

? Non-comparability is transitive: for all x, y, z if $x\ N\ y$ and $y\ N\ z$ then $x\ N\ z$.