# PHPE 400 <br> Individual and Group Decision Making 

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## Rational choice

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A decision maker chooses rationally if her preferences are rational and there is nothing available that the decision maker prefers to what she chooses.

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## Mathematically describing preferences

notes.phpe400.info/mathematical-preliminaries/sets.html notes.phpe400.info/mathematical-preliminaries/relations.html

Answer the mathematical notation quiz on Tophat before your discussion section on Friday (the answers will be discussed during sections):
https://app.tophat.com/e/384276/content/1117900::
f6a2a05b-dd5c-44fd-9297-fd322cfab11a?open_fullscreen= true

## Mathematical background: Relations

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E.g., $X=\{a, b, c, d\}, R=\{(a, a),(b, a),(c, d),(a, c),(d, d)\}$

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Suppose that $X$ is a set. A relation on $X$ is a set of ordered pairs from $X$ : $R \subseteq X \times X$.

Example: $X=\{a, b, c, d\}, R=\{(a, a),(b, a),(c, d),(a, c),(d, d)\}$


$$
\begin{aligned}
& a R a \\
& b R a \\
& c R d \\
& a R C \\
& d R d
\end{aligned}
$$

## Representing Preferences

Let $X$ be a set of outcomes. A decision maker's preference over $X$ is represented by relations on $X$ :

- $P \subseteq X \times X$ where $a P b$ means that the decision maker strictly prefers $a$ to $b$.


## Representing Preferences

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- $I \subseteq X \times X$ where $a$ I $b$ means that the decision maker is indifferent between $a$ and $b$.
- $N \subseteq X \times X$ where $a \mathrm{Nb}$ means that the decision maker cannot compare $a$ and $b$.


## Strict Preference, I

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A decision maker's strict preference over a set $X$ is represented as a relation $P \subseteq X \times X$.

The underlying idea is that if $P$ represents the decision maker's strict preference and $x P y$ (i.e., the decision maker strictly prefers $x$ to $y$ ), then the decision maker would pay some non-zero amount money to trade $y$ for $x$.

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The underlying idea is that if $P$ represents the decision maker's strict preference and $x P y$ (i.e., the decision maker strictly prefers $x$ to $y$ ), then the decision maker would pay some non-zero amount money to trade $y$ for $x$.
$P$ is asymmetric: for all $x, y \in X$, if $x P y$, then it is not the case that $y P x$ (written not-y $P x$ ).

## Indifference/Incommensurable

Suppose that $P$ is an asymmetric relation on $X$ (interpreted as a decision maker's strict preference). Suppose that $x, y \in X$ with not- $x P y$ and not- $y P x$.

## Indifference/Incommensurable

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Suppose that $P$ is an asymmetric relation on $X$ (interpreted as a decision maker's strict preference). Suppose that $x, y \in X$ with not- $x P y$ and not- $y P x$. There are two reasons why this might hold:

1. The decision maker is indifferent between $x$ and $y$. In this case, we write $x$ I $y$.
2. The decision maker cannot compare $x$ and $y$. In this case, we write $x N y$.

## Preferences

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There are four distinct ways a decision maker can compare $x$ and $y$ :

1. $x P y$ : the decision maker strictly prefers $x$ to $y$.
2. $y P x$ : the decision maker strictly prefers $y$ to $x$.
3. $x$ I $y$ : the decision maker is indifferent between $x$ and $y$.
4. $x N y$ : the decision maker cannot compare $x$ and $y$.

## Symmetric/Asymmetric Relations

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Symmetric relation: for all $x, y \in X$, if $x R y$, then $y R x$
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Irreflexive relation: for all $x \in X$, if not $-x$ R

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E.g., $X=\{a, b, c, d\}$


## Preferences - Minimal Constraints

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A decision maker's preferences on $X$ is represented by three relations $P \subseteq X \times X, I \subseteq X \times X$ and $N \subseteq X \times X$ satisfying the following minimal constraints:

1. For all $x, y \in X$, exactly one of $x P y, y P x, x I y$ and $x N y$ is true.
2. $P$ is asymmetric
3. I is reflexive and symmetric.
4. $N$ is symmetric.

A decision maker's preferences are rational when they are transitive and complete.

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## Transitive Relations

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## Transitivity

Strict preference is transitive: for all $x, y, z$ if $x P y$ and $y P z$ then $x P z$

## Transitivity

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Strict preference is transitive: for all $x, y, z$ if $x P y$ and $y P z$ then $x P z$
? Indifference is transitive: for all $x, y, z$ if $x I y$ and $y I z$ then $x I z$
? Non-comparability is transitive: for all $x, y, z$ if $x N y$ and $y N z$ then $x N z$.

